

# $H_{\infty}$ Control of Container Yard's Supply Chain System

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## Abstract

**This paper is about the problem of  $H_{\infty}$  control for container yard's supply chain system. With the uncertain problem in the process of port yard goods export, the system is modeled as a switching system according to the different states of the yard capacity level under the consideration of the cargo's attenuation factor. The controller design and performance analysis of the system were applied in switching model, and the bullwhip effect was effectively suppressed under the condition of ensuring the steady of the container yard's supply chain system with  $H_{\infty}$  performance. Simulation examples show the effectiveness of the results.**

## Keywords

**Dual-rate sampling;  $H_{\infty}$  robust control; Container yard's supply chain system.**

## 1. Introduction

With the development of foreign trade and transportation industry, the throughput of containers develops rapidly [1, 2]. The large increase of container loading and unloading in the yard also brings higher requirements to control the reasonable distribution of storage space in the yard [3, 4, 5]. Especially in some container terminals that are short of port space resources, when a large number of containers are loaded and unloaded, there is no reasonable and efficient storage yard allocation plan [6, 7], which will cause congestion in container terminals and further affect the implementation of other operations of the terminals, reducing the working efficiency of the terminals. Therefore, it is extremely important to reasonably allocate the resources of the front yard and the rear yard. However, most of the existing researches have improved the allocation mechanism from the algorithm [8, 9, 10]. Paper [11] Optimize space for storage yard considering yard inventory forecasts. Paper [12] improve the performance of the yard storage by minimizing handling time of export containers. None of them can regard the export of containers and the distribution and transfer of front and rear yards as a dynamic overall system [13]. The front yard is located between the Wharf and the rear yard. It is a place for stacking containers to accelerate the loading and unloading efficiency of ships. Its main function is: before the ship arrives at the port, piles up in advance to load the exported container; Temporarily load the imported containers during unloading. The rear yard is located at the rear of the port. It mainly stocks exported containers that cannot be immediately loaded onto the ship or imported containers that cannot be immediately shipped out of the port. When the exported goods arrive at the port, some containers not in a hurry to be loaded are temporarily stored in the rear yard, then transferred to the front yard, and then loaded out of the port. This process can be regarded as a closed-loop supply chain system and the feedback control theory has been applied to such supply chain system and can effectively reduce the bullwhip effect. Bullwhip effect refers to a phenomenon of demand variation and amplification in the supply chain. It means that when the information flow is transmitted from the end client to the original supplier, the information sharing cannot be effectively realized. As a result, the information is distorted and amplified step by step, leading to more and more fluctuations in demand

information. Reducing the bullwhip effect can be seen as a  $H^\infty$  control problem [14, 15]. The closed-loop supply chain generates the volume of containers in the rear yard and the volume of containers transferred from the rear yard to the front yard  $u(k)$  through the storage level of the yard, suppressing the disturbance  $\omega(k)$  of the uncertain demand of the system so that obtain the ideal minimum operating cost  $y(k)$  of the whole system. This inhibitory level can be described by

$$\frac{\|y\|_2}{\|\omega\|_2} \leq \gamma \tag{1}$$

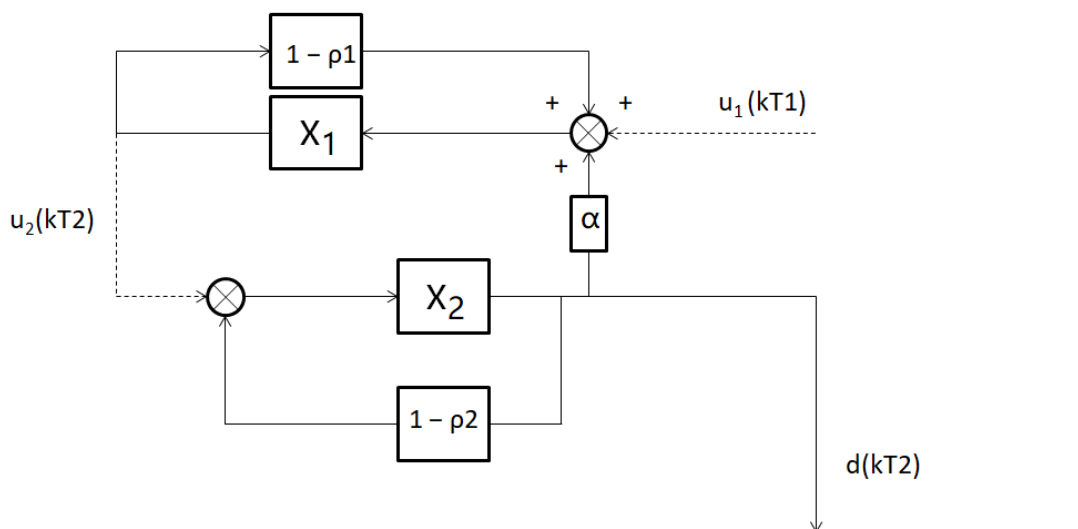
The formula (1) shows that the smaller  $\gamma$  is, the better the performance of the system is, it essentially reflects the degree to which the system inhibits the bullwhip effect. Therefore, it is great significance to apply the  $H^\infty$  control to study the uncertainty of closed-loop supply chain.

## 2. Modeling Container Yard's Supply Chain System

Consider a container yard's supply chain system of two facilities, One is the front yard and one is the rear yard, where the rear yard stores the exported containers that not in a hurry to be loaded, and then the exported containers transferred to the front yard, finally loaded out of the port. The diagram of such a system is shown in Figure 1, where the freight flow and the information flow are denoted by the solid line and the dash line respectively. It is assumed that two yards have different sampling periods, which are denoted by  $T_1$  and  $T_2$  respectively, and satisfy  $T_1 = l_1h$  and  $T_2 = l_2h$  [16] ( $l_1, l_2 \in \mathbb{N}^+$  are relatively prime,  $h$  is the base period). So the whole system, which is the period after lifting, can be calculated as  $T = l_1l_2h$ . Where  $x_1(kT_1) \in \mathbb{R}$  denote the capacity of the rear yard,  $x_2(kT_2) \in \mathbb{R}$  denote the capacity of the front yard,  $u_1(kT_1) \in \mathbb{R}$  and  $u_2(kT_2) \in \mathbb{R}$  respectively denote the transfer rate of two yards at time  $k$ , then  $d(kT_2) \in \mathbb{R}$  denote the load rate and satisfy

$$d(kT_2) = \bar{d} + \omega(kT_2) \tag{2}$$

Where  $\bar{d}$  denote the known quantity of loading and  $\omega(kT_2)$  denote the uncertainty demand.



**Figure 1.** Block diagram of two-time-scale container yard's supply chain system

For system modeling, we need the following assumptions.

**Assumption 1:** The value of container goods in two yards may deteriorate with time, let  $\rho_1$  and  $\rho_2$  respectively denote the deteriorate rate of the rear yard and the front yard.

**Assumption 2:** The yards have certain storage capacity, let  $c_1$  and  $c_2$  respectively denote the maximum capacities of the rear yard and the front yard.

**Assumption 3:** If the ship cannot arrive at the port on time due to temporary weather or human reasons, and the container cannot be shipped on time, in order to relieve the pressure of the front yard, the delayed container shall be sent back to the rear yard, let  $\alpha$  denote the average percentage of containers that be sent back to the rear yard.

**Remark 1:** Assumptions 1 and 2 are standard assumptions for container yard's supply chain system. There is only limited space in the yard to store a certain amount of container goods, and the container goods will devalue or deteriorate, especially for perishable products.

Considering the rear yard, when  $0 < x_1(kT_1) \leq C_1$ , the yard model is described by

$$x_1[(k + 1)T_1] = (1 - \rho_1)x_1(kT_1) + u_1(kT_1) - u_2(kT_2) + \alpha(1-\rho_2) x_2(kT_1) \tag{3}$$

When  $x_1(kT_1) \leq 0$ , it means that no containers in the rear yard can be transferred to the front yard immediately. Therefore, the model is

$$x_1[(k + 1)T_1] = x_1(kT_1) + u_1(kT_1) - u_2(kT_2) + \alpha(1-\rho_2)x_2(kT_1) \tag{4}$$

Similarly, for the front yard, the models for  $0 < x_2(kT_2) \leq c_2$  and  $x_2(kT_2) \leq 0$  are

$$x_2[(k + 1)T_2] = (1 - \rho_2)x_2(kT_2) + u_2(kT_2) - \alpha(1-\rho_2) x_2(kT_2) - d(kT_2) \tag{5}$$

$$x_2[(k + 1)T_2] = x_2(kT_2) + u_2(kT_2) - d(kT_2) \tag{6}$$

**Remark 2:** As shown in Figure 1,  $u_2(kT_2)$  connects between the front yard and the rear yard. It denotes the amount of containers be sent from the rear yard to the front yard.

By considering a container yard's supply chain system consisting of two yards with Dual-echelon control, a switched system with subsystems (3)–(6) is constructed. As for the switched system with different running time scales, we can improve the subsystems (3)–(6).

From (3), we can obtain the following model for  $0 < x_1(kT_1) \leq c_1$ :

$$\begin{aligned} x_1[(k + 1)T] &= x_1[(kT + T)] \\ &= (1-\rho_1)^{l_2}x_1(kT) + (1-\rho_1)^{l_2-1}u_1(kT) + (1-\rho_1)^{l_2-2}u_1(kT+T_1) + \dots + (1-\rho_1)u_1(kT+(l_2-2)T_1) + \\ &u_1(kT+(l_2-1)T_1) - (1-\rho_1)^{l_1-1}u_2(kT) - (1-\rho_1)^{l_1-2}u_2(kT+T_2) - \dots - (1-\rho_1)u_2(kT+(l_1-2)T_2) - \\ &u_2(kT+(l_1-1)T_2) + \alpha(1-\rho_2)(1-\rho_1)^{l_2-1} x_2(kT) + \alpha(1-\rho_2)(1-\rho_1)^{l_2-2}x_2(kT+T_1) + \dots + \\ &\alpha(1-\rho_2)(1-\rho_1)x_2(kT+(l_2-2)T_1) + \alpha(1-\rho_2)(kT+(l_1-1)T_1) \\ &= (1-\rho_1)^{l_2}x_1(kT) + u_1(kT) - u_2(kT) + \alpha(1-\rho_2)x_2(kT) \end{aligned} \tag{7}$$

Where

$$u_1(kT) = (1-\rho_1)^{l_2-1}u_1(kT) + (1-\rho_1)^{l_2-2}u_1(kT+T_1) + \dots + (1-\rho_1)u_1(kT+(l_2-2)T_1) + u_1(kT+(l_2-1)T_1) \tag{8}$$

$$u_2(kT) = (1-\rho_1)^{l_1-1}u_2(kT) - (1-\rho_1)^{l_1-2}u_2(kT+T_2) - \dots - (1-\rho_1)u_2(kT+(l_1-2)T_2) - u_2(kT+(l_1-1)T_2) \tag{9}$$

$$\alpha(1-\rho_2)x_2(kT) = \alpha(1-\rho_2)(1-\rho_1)^{l_2-1}x_2(kT) + \alpha(1-\rho_2)(1-\rho_1)^{l_2-2}x_2(kT+T_1) + \dots + \alpha(1-\rho_2)(1-\rho_1)x_2(kT+(l_2-2)T_1) + \alpha(1-\rho_2)x_2(kT+(l_1-1)T_1) \tag{10}$$

Then, according to (4), (8), (9), and (10), a simple model is given for  $x_1(kT_1) \leq 0$

$$x_1[(k+1)T] = x_1(kT) + u_1(kT) - u_2(kT) + \alpha(1-\rho_2)x_2(kT) \tag{11}$$

Using the same method, the model for  $0 < x_2(kT_2) \leq c_2$  is given as

$$x_2[(k+1)T] = (1-\rho_2)^{l_1}x_2(kT) + u_2(kT) - \alpha(1-\rho_2)x_2(kT) - d(kT) \tag{12}$$

Where

$$d(kT) = (1-\rho_2)^{l_1-1}d_2(kT) + (1-\rho_2)^{l_1-2}d_2(kT+T_2) + \dots + (1-\rho_2)d_2(kT+(l_1-2)T_2) + d_2(kT+(l_1-1)T_2) \tag{13}$$

and for  $x_2(k) \leq 0$  the model is

$$x_2[(k+1)T] = x_2(kT) + u_2(kT) - d(kT) \tag{14}$$

Remark 3: when  $l_1=l_2$ , the two-time-scale container yard's supply chain system changes to be a single-rate system.

let the output of the whole system be  $y(k) = Cx(k)$ , where  $x(k) = [x_1(k) \ x_2(k)]^T$  and C is a cost parameter matrix with appropriate dimension, and it is assumed to be invertible.

Considering (7) and (12), we can get the following supply chain system:

$$\begin{aligned} x(k+1) &= A_1x(k) + b + B_1u(k) + B_2\omega(k) \\ y(k) &= Cx(k) \end{aligned} \tag{15}$$

Where

$$A_1 = \begin{pmatrix} (1-\rho_1)^{l_2} & \alpha(1-\rho_2) \\ 0 & (1-\rho_2)^{l_1} - \alpha(1-\rho_2) \end{pmatrix} \quad u(k) = \begin{pmatrix} u_1(k) \\ u_2(k) \end{pmatrix} \quad b = \begin{pmatrix} 0 \\ -d \end{pmatrix} \quad \omega(k) = \begin{pmatrix} 0 \\ \omega(k) \end{pmatrix} \quad B_1 = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \quad B_2 = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$$

Considering (7) and (14), we can get:

$$\begin{aligned} x(k+1) &= A_2x(k) + b + B_1u(k) + B_2\omega(k) \\ y(k) &= Cx(k) \end{aligned} \tag{16}$$

Where

$$A_2 = \begin{pmatrix} (1-\rho_1)^{l_2} & \alpha(1-\rho_2) \\ 0 & 1 \end{pmatrix}$$

Considering (11) and (12), we can get:

$$x(k+1) = A_3x(k) + b + B_1u(k) + B_2\omega(k)$$

$$y(k) = Cx(k) \tag{17}$$

Where

$$A_3 = \begin{pmatrix} 1 & \alpha(1 - \rho_2) \\ 0 & (1 - \rho_2)^{l_1} - \alpha(1 - \rho_2) \end{pmatrix}$$

Considering (11) and (14), we can get:

$$\begin{aligned} x(k+1) &= A_4x(k) + b + B_1u(k) + B_2\omega(k) \\ y(k) &= Cx(k) \end{aligned} \tag{18}$$

Where

$$A_4 = \begin{pmatrix} 1 & \alpha(1 - \rho_2) \\ 0 & 1 \end{pmatrix}$$

In the above derivation, because of the uncertainty of the cargo export of container ports, the uncertain inventory level of the front yard and the rear yard is affected, and subsystems of four states are generated.

According to the (19), the supply chain system (15)-(18) can translate into the following switching system:

$$\begin{aligned} x(k+1) &= A_i x(k) + b + B_1 u(k) + B_2 \omega(k) \\ y(k) &= C x(k) \end{aligned} \tag{19}$$

### 3. Robust Control Analysis

Considering the inventory capacity of the front yard and the rear yard, as well as the dynamic transfer process between the two yards, the equation (19) is given. Through linear matrix inequality (LMI), we transform it into robust  $H_\infty$  control problem.

Theorem 1: container yard's supply chain system (19) is exponentially stable with a prescribed  $H_\infty$  performance level  $\gamma$ , if there exist symmetric positive definite matrices  $P > 0$ ,  $Q > 0$ , the following linear matrix inequalities is solvable:

$$\Omega = \begin{pmatrix} A_i^T P A_i + Q + C^T C & A_i^T P B_1 K_i & A_i^T P B_2 \\ * & K_i^T B_1^T P B_1 K_i & K_i^T B_1^T P B_2 \\ * & * & B_2^T P B_2 - \gamma^2 I \end{pmatrix} < 0 \tag{20}$$

Theorem 1 is a prerequisite for solving the robust H control problem of this system. Its principle is to transform the bullwhip effect describing the container yard's supply chain system into a robust  $H_\infty$  control problem.

Theorem 2: container yard's supply chain system (19) is exponentially stable with a prescribed  $H_\infty$  performance level  $\gamma$ , if there exist symmetric positive definite matrices  $\bar{P} > 0$ ,  $\bar{Q} > 0$ , the following linear matrix inequalities is solvable:

$$\Omega = \begin{bmatrix} -\bar{P} & 0 & 0 & \bar{P}A_i^T & \bar{P} & \bar{P} \\ * & -\bar{Q} & 0 & X_i^T B_1^T & 0 & 0 \\ * & * & -\gamma^2 I & B_2^T & 0 & 0 \\ * & * & * & -\bar{P}^{-1} & 0 & 0 \\ * & * & * & * & -\bar{Q} & 0 \\ * & * & * & * & * & -(C^T C)^{-1} \end{bmatrix} < 0 \tag{21}$$

Where the control gains  $K_i$  can be calculated as  $K_i = X_i \bar{Q}^{-1}$

### 4. Simulation Analysis

In order to directly confirm the effectiveness of our proposed strategy, we perform simulation analysis through MATLAB here. Based on experience, it is assumed that the deteriorate rate of the rear yard and the front yard are  $\rho_1 = 0.04$  and  $\rho_2 = 0.02$ , the average percentage  $\alpha$  of containers that be sent back to the rear yard is 0.05, the base period  $h = 0.01$ , the sampling periods  $T_1$  and  $T_2$  are 0.04 and 0.05, so the frame period  $T = 0.20$ ,  $d=0$ ,

$$C = \begin{pmatrix} 0.3 & 0 \\ 0 & 0.3 \end{pmatrix}$$

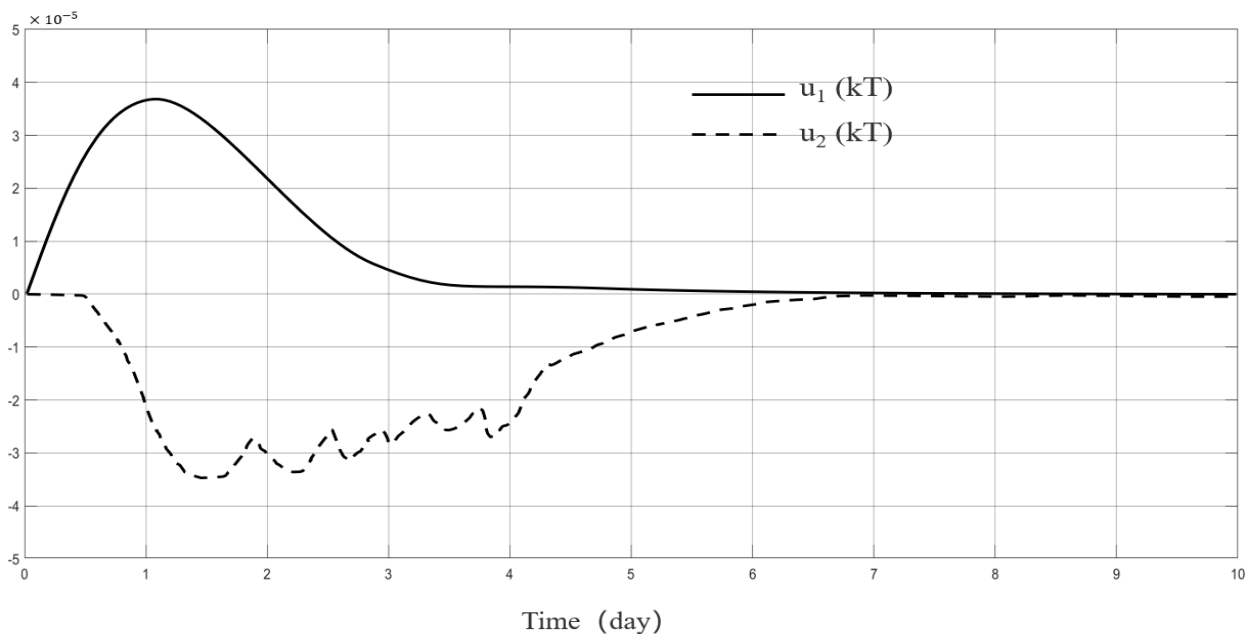
By solving (21) with the mincx solver in MATLAB LMI Toolbox, the control gain can be obtained as:

$$K_1 = \begin{bmatrix} 0.2536 & 0 \\ 0 & 0.2428 \end{bmatrix} \times 10^{-6} \quad K_2 = \begin{bmatrix} 0.2951 & 0 \\ 0 & 0.2813 \end{bmatrix} \times 10^{-6}$$

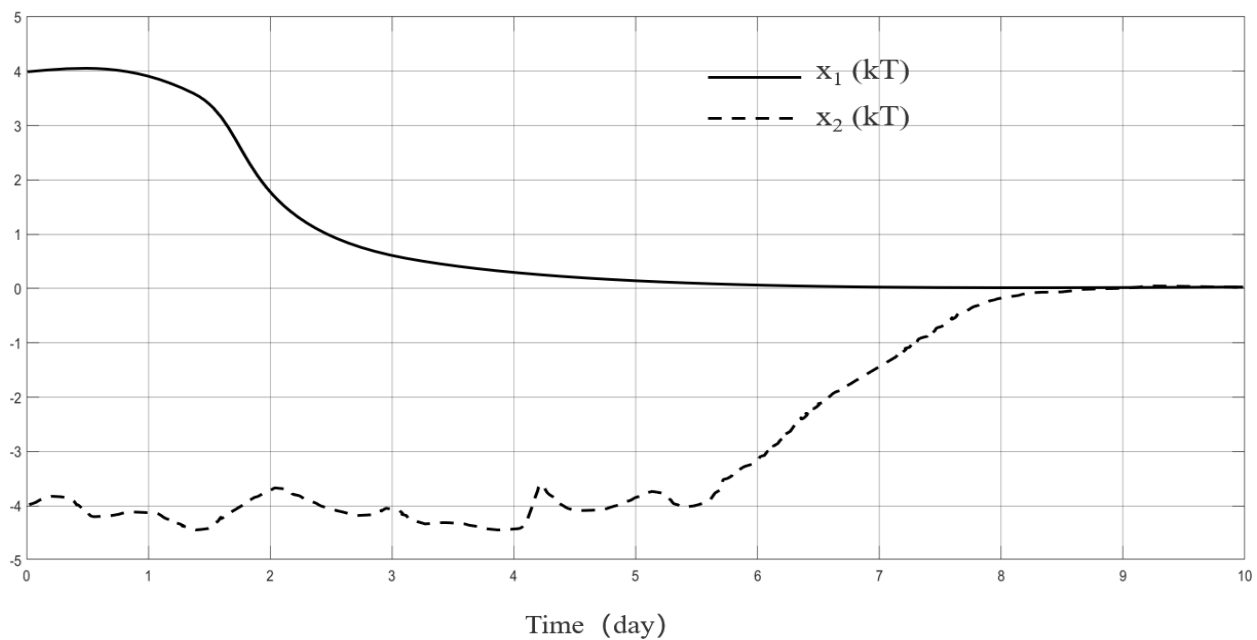
$$K_3 = \begin{bmatrix} 0.3134 & 0 \\ 0 & 0.3053 \end{bmatrix} \times 10^{-6} \quad K_4 = \begin{bmatrix} 0.3284 & 0 \\ 0 & 0.3169 \end{bmatrix} \times 10^{-6}$$

and  $\gamma_{\min}$  is 3.4875.

Set the initial condition  $x(k)$  is  $[4 \ -4]$ , we can obtain the Figure 2 and Figure 3 by Simulink in MATLAB.



**Figure 2.** Container transfer level of the yard



**Figure 3.** The inventory levels of the front yard and the rear yard

In the Figure 2, due to uncertain demand and human and environmental factors, the container transportation flows  $u_1$  and  $u_2$  make large fluctuations, but with our subsystem switching strategy and dual-rate sampling strategy,  $u_1$  and  $u_2$  gradually stabilized and tended to 0, and  $u_1$  flattens out is faster than  $u_2$ .

In the Figure 3, the inventory level of the front yard quickly stabilized. Although the inventory level of the rear yard fluctuated slightly at the beginning, it also stabilized afterwards. In addition, the inventory levels of the front yard and the rear yard are both close to 0, indicating that there is no inventory pressure at the two yards, and the transportation efficiency of the containers is very good.

Combining Figure 2 and Figure 3, we can find that the bullwhip effect has a very small impact on this container yard's supply chain system. The two storage yards have shown better inventory and transfer capabilities due to our proposed control strategy, which will greatly reduce inventory costs, maintenance costs and a series of costs, the stability of the container yard's supply chain system and economic benefits have been significantly improved.

## 5. Conclusion

In this paper, a container yard's supply chain system with dual-rate sampling is constructed. According to the inventory levels of the front yard and the rear yard, we listed out four situations as well as modelled respectively, and the holistic system will switch among these four subsystems. When the system satisfies the exponential stability in the sense of Lyapunov, the dynamic performance of the system is judged by transforming the bullwhip effect problem into a robust  $H_\infty$  control problem. Finally, the simulation proves that under the control strategy we proposed, the bullwhip effect has little impact on the container yard's supply chain system, and the inventory level can quickly stabilize.

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