

# Hybrid Simulated Annealing Algorithm for Vehicle Routing Problem

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## Abstract

Based on the classical simulated annealing algorithm, using natural number coding method, a local search method for sub-paths is designed for the weakness of fine tuning ability and local search ability. Re-positioning the customers in each vehicle path through an adaptive local search method to achieve fine tuning of the current solution and enhance local search capabilities. The traditional annealing algorithm and the hybrid- annealing algorithm are tested by using international standard test cases. Compared with the iterative local search algorithm (ILS) and the quantum evolution algorithm and artificial bee colony algorithm, the experimental results show that the hybrid simulated annealing algorithm is more effective.

## Keywords

Vehicle path problem; Annealing algorithm; Local search.

## 1. Introduction

In recent years, the logistics industry has developed rapidly. In 2018, China's social logistics totaled 283.1 trillion RMB, a year-on-year increase of 6.4%. However, the refined management and operational efficiency of China's logistics are still at a relatively low level. China's logistics costs accounted for 14.6% of GDP, which was 1.9 times that of the United States during the same period. The gap between China and developed countries is obvious. Therefore, how to improve the overall operational efficiency of China's logistics and reduce costs is an urgent problem that needs to be resolved.

Vehicle Routing Problem (VRP) is an important problem of refined management in the logistics field, and it is a common concern in the field of scientific research and logistics transportation companies. With the advancement of computer science, a variety of solutions to the VRP problem and its deformation problems have been proposed and developed. Since the VRP problem has been proved to be NP-hard, it is not easy to obtain the optimal solution for the VRP problem in real life. Arnold and Sorensen [1] believe that the heuristic algorithm provides the best balance between solution quality and calculation time, which is conducive to logistics companies to deliver goods in a cost-effective manner and to meet the time requirements of customer service. In order to obtain high-quality feasible solutions to the VRP problem, researchers have proposed iterative local search algorithms, quantum evolutionary algorithms, genetic algorithms, ant colony algorithms, and bee colony algorithms. Literature [2] uses the ILS algorithm with improved disturbance mechanism to solve the CVRP problem; Literature [3] uses the domain inversion operation method to improve the artificial bee colony algorithm to solve the vehicle understanding problem; Literature [4] uses genetic algorithm and tabu search algorithm A hybrid algorithm is used to solve the VRP problem, which uses a genetic algorithm to perform a global search, and then uses a tabu search algorithm to perform a deep search for the optimal population. Literature [5] uses quantum evolution algorithm to solve the CVRP problem, introduces quantum revolving gate to achieve evolution, uses catastrophe mechanism

to ensure the diversity of the solution, and then combines 2-otp deep optimization through nearest neighbor insertion. Literature [6] uses the empire competition algorithm to solve the OVRP problem. The empire competition algorithm is a social heuristic algorithm, which is solved by simulating the social phenomenon of empire colonization. Literature [7] uses a hybrid algorithm combining improved ant colony optimization algorithm and multi-domain descent search to solve this problem. These algorithms have been widely used in vehicle routing problems.

In this paper, the simulated annealing algorithm is applied to the vehicle routing problem, and the sub-route local search operator (Inter-route Relocation) is designed to enhance the local fine-tuning ability and local optimization ability of the algorithm, improve the search efficiency of the algorithm, and let the algorithm solve the quality and computational efficiency A good balance is achieved between. Experiments on the international benchmark test case set show that compared with the conventional simulated annealing algorithm and some optimization algorithms proposed in recent years, the solution quality of the algorithm in this paper has been greatly improved, and the algorithm has good practical application value.

Organization of the Text

### 1.1. Problem Description and Definition

TheVRP problems can be subdivided into multiple subtypes. This paper studies the typical Capacitated Vehicle Routing Problem (CVRP), that is, there are K vehicles with a load capacity of Q in the warehouse, and n distribution points are distributed from the warehouse. After the distribution is completed, they return to the warehouse. Under the premise of meeting the demand of freight, the shortest route plan of the vehicle travel route is solved.

Problem assumptions: (1) The distribution task of each distribution point has one and only one vehicle service, and it is only served once; (2) All vehicles depart from the warehouse and return to the warehouse after completing the distribution task. (3) The total demand of all the distribution points served by each vehicle does not exceed the load Q of the vehicle.

According to the hypothesis, the warehouse number is 0 and the distribution point number is {1, 2, ...,n}, then the vertex set is  $V=\{0, 1, 2, \dots,n\}$ . The demand of each distribution point is  $q_i$ ( $i=0, 1, 2, \dots, n$ ), and  $q_i \leq Q, q_0 = 0$ . The distance (cost) between delivery points i and j is denoted  $asc_{ij}$ . Define the decision variables as:

$$x_{ijk} = \begin{cases} 1, \text{Vehicle } k \text{ drives from delivery point } i \text{ to delivery point } j \\ 0, \text{other} \end{cases}$$

$$y_{ik} = \begin{cases} 1, \text{Vehicle } k \text{ drives from delivery point } i \text{ to delivery point } j \\ 0, \text{other} \end{cases}$$

The mathematical expression for constructing the optimization model is as follows:

$$Min.Z = \sum_{i=0}^n \sum_{j=0}^n \sum_{k=1}^K c_{ij} x_{ijk} \tag{1}$$

$$s.t. \sum_{k=1}^K y_k = 1, \forall i \in \{1, 2, B, n\} \tag{2}$$

$$\sum_{i \in V} x_{ijk} = y_{ik}, \forall j \in V, k \in \{1, 2, B, K\} \tag{3}$$

$$\sum_{j \in V} x_{ijk} = y_{ki}, \forall i \in V, k \in \{1, 2, B, K\} \quad (4)$$

$$\sum_{i \in V} y_{ik} q_i \leq Q, \forall i \in V \quad (5)$$

In the above model, (1) is the optimization goal, that is, the minimum travel distance (operating cost); constraint (2) restricts each delivery point to one and only one car service; constraint (3) and constraint (4) guarantee each delivery point has one and only one service; constraint (5) ensures that the load of the vehicle does not exceed the capacity of the vehicle.

## 2. Algorithm Design

### 2.1. Hybrid Simulated Annealing Algorithm

Although the traditional simulated annealing algorithm has the advantages of fewer parameters, simple principle, flexible use, and suitable for solving the global optimal or approximate global optimal solution of the optimization problem, it requires a sufficiently high initial temperature, a slow annealing speed, and a relatively high initial temperature. High number of iterations, and the number of disturbances at the same temperature. This also leads to the efficiency of traditional SA algorithm can not meet the needs of real logistics enterprises. Therefore, we propose to increase the local fine-tuning ability and local search ability of the algorithm by adding the sub-route local search optimization operator (Inter-route Relocation), and improve the convergence speed of the algorithm and the operation efficiency of the algorithm.

The algorithm designed in this paper as a whole includes three main steps: initial solution generation, simulated annealing, and sub-path local search. The execution process is as follows:

- (1) Construct the initial solution  $S_0$  of the problem by the natural number coding method [6]; let  $S^*$  be the global optimal solution. Initially set:  $S^* = S_0$ .
- (2) Using simulated annealing algorithm to improve the initial solution  $S_0$ , obtain the local optimal solution  $S_1$ , and set  $S^* = S_1$ .
- (3) Use the local search operator to optimize the sub-path of each vehicle service in  $S_1$  and obtain a new local optimal solutions  $S_1^*$ .
- (4) Go to step (2) to continue execution until the loop termination condition is met.
- (5) Output the global optimal solution  $S^*$ .

In this paper, the SA algorithm builds a sub-route local search algorithm (Inter-route Relocation) by reducing the initial temperature of the serial simulated annealing algorithm and the number of iterations at the same temperature to accelerate the convergence speed of the algorithm. In many cases, by adding a local search after mutation and recombination or merging the local search process into a recombination operator, the algorithm performance of the combination problem can be significantly improved. The sub-route local search operator starts when the SA algorithm falls into the local minimum in stages, and fine-tunes the current scheme to realize the deep search in the local domain at this stage [18]. This method can improve the convergence speed of the SA algorithm and can produce a high-quality solution in a reasonable time.

According to the flow of the improved hybrid simulated annealing algorithm, the pseudo code for solving the VRP problem is as follows:

```

1.       $s := s_0, T_0 := \gamma \times \text{cost}(s_0)$ 
2.       $k := 0, \omega := 0, n := 0$ 
3.      WHILE ( $m < M$ ) DO
4.      WHILE( $\omega < \omega_{max}$ ) DO
5.           $l := 0, \text{updateOptimalSolution} := \text{true}$ 
6.          WHILE( $l < L$ ) DO
7.              BEGIN
8.                   $s'_k = \text{local search}(s_k) s'_k \in N(s_k)$ 
9.                  IF  $\text{cost}(s'_k) < \text{cost}(s_k)$ , THEN
10.                      $s_k := s'_k, \text{updateOptimalSolution} := \text{true}$ 
11.                 ELSE
12.                      $s_k := \hat{s}_k$  with probability  $e^{\frac{\text{cost}(s_k) - \text{cost}(\hat{s}_k)}{T_k}}$ 
13.                      $l := l + 1$ 
14.                 END IF
15.             END WHILE
16.              $k := k + 1, T_{k+1} := \beta \times T(k)$ 
17.             IF ! $\text{updateOptimalSolution}$ 
18.                  $\omega := \omega + 1$ 
19.             END WHILE
20.             Inter – route Relocation( $s_k$ )
21.              $m := m + 1$ 
22.         END WHILE
23.     Return  $s_k$ 

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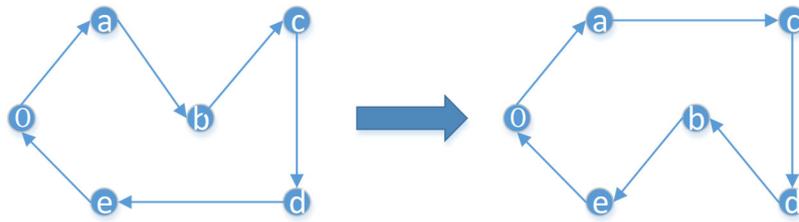
Among them:  $\text{cost}(s_k) = \sigma c_d (c \times n + e_{min}) + (1 - \sigma)c_t d$ ,  $T_k = \gamma \times \text{cost}(s_k)$ ;  $s_k$  is the feasible solution;  $d$  is the total travel distance of the path;  $n$  is the number of customers ;  $\gamma < 1$  and a constant;  $\sigma$  is a constant, weighing the distribution cost and driving cost parameters,  $\sigma \in [0,1]$ ;  $c_d$  is the distribution cost of vehicle  $k$ ;  $e_{min}$  is the number of customers in the shortest path;  $c_t$  is the unit distance Driving cost;  $\beta$  is the cooling rate;  $\beta < 1$  and constant;  $c$  is the number of vehicles used under the current solution;  $l$  is the length of the Markov chain;  $\omega$  is the counter of the current temperature during the continuous decrease of temperature.

## 2.2. Sub-path Local Search Operator

Based on the process and improvement ideas of the hybrid simulated annealing algorithm, this article aims at the traditional simulated annealing algorithm that requires a sufficiently high initial temperature, a slow annealing speed, a high number of iterations, a large number of disturbances at the same temperature, and the timeliness of practical applications. The contradiction between the requirements of the sub-path local search operator is designed. The sub-path local search operator coordinates the simulated annealing algorithm to achieve the best balance between solution quality and calculation time, which is beneficial for logistics companies to deliver goods in a cost-effective manner and meet the time requirements of customer service.

When the I-SA algorithm in this paper falls into a local minimum, it will no longer use high temperature and a long Markov chain to break through the current local optimal solution, but will be optimized by the sub-route local search operator (Inter-route Relocation) To achieve a breakthrough in the local minimum, thereby improving the convergence speed of the I-SA algorithm, enhancing the local fine-tuning ability of the algorithm, and searching for the minimum in each local area as much as possible. After the I-SA algorithm obtains a local minimum value, the customer in each vehicle path changes the position (the customer changes the position of the same path, and the total vehicle load does not change), thereby minimizing the driving distance. For example: from the path (0, a), (a, b), (b, c), (c, d), (d, e), (e, 0), by

relocating b, in order to Reduce the travel distance of the current path [15]. Figure 1 shows the process of relocating customers within the same vehicle route.



**Figure 1.** Sub-path internal exchange

The sub-path local domain search performs a local search in each vehicle path of the current solution  $S_k$  to generate a new feasible solution  $S'_k$ . This article will use 2-opt and Or-opt two local domain search methods through an adaptive method.

(1) 2-opt method: reduce the path distance by changing the order of customers in each path. If the path distance cost is reduced, the improved path is retained; on the contrary, the path remains unchanged from the original path.

(2) Or-opt method: Relocate in the path by assigning n in the path to consecutive client nodes.

The pseudo code of the Inter-route Relocation operator is as follows:

1. **input: a feasible solution  $S_k$**
2.  $NS_k = S_k; \rho = (1, 1); K$
3. **If  $n \leq K$ :**
4.  $s^n = NS_k^n$
5. **repeat**
6. **Select methods  $d \in \Omega$  using  $\rho$**
7.  $X^t = d(s^n);$
8. **If  $C(X^t) < C(s^n)$  then**
9.  $s^n = X^t$
10. **end if**
11. **update  $\rho$**
12. **Until stop criterion is met**
13.  $NS_k^n = s^n$
14. **end if**
15. **return  $NS_k^n$**

Where  $\Omega$  represents the set of two local domain search methods, 2-opt and Or-opt, K is the total number of service vehicles, and  $\rho$  represents the weight set of the two methods. The calculation formula of the selection probability is:

$$\phi_j = \frac{\rho_j}{\sum_{k=1}^{|\Omega|} \rho_k}$$

The greater the weight of the algorithm, the greater the probability of being selected, and the size of the weight is dynamically adjusted according to the performance of the 2-opt and Or-opt methods in the search process. After an iteration is completed, the following function is used to adjust according to the iteration results Its weight:

$$\psi = \max \begin{cases} \omega_1 & \text{If the new solution is better than the current solution} \\ \omega_2 & \text{If the new solution is rejected} \end{cases}$$

Where  $\omega_1 \geq \omega_2 \geq 0$ , if  $a$  is the method used in the last iteration, then its weight is updated as follows:

$$\rho_a = \lambda\rho_a + (1 - \lambda)\psi$$

### 3. Algorithm Performance Analysis and Comparison

#### 3.1. Algorithm Test

The parameters of the algorithm are set as follows: the number of iterations  $M$  is 100, the initial temperature is 10, the Markov Chain length is 30,  $\sigma$  is 1, the temperature and cost relationship constant  $\gamma=1$ , and the cooling constant  $\beta=0.99$ .

In order to compare the performance before and after the algorithm is improved, the traditional SA algorithm and the improved algorithm (I-SA) proposed in this paper are used for testing. The SA and I-SA parameter settings are the same, and only the sub-route local search algorithm (Inter-route Relocation) is added to the latter. Among them, the traditional SA algorithm also uses a multi-stage iterative method to solve the problem.

Using traditional SA algorithm and I-SA algorithm to solve the test case set is the internationally public standard CVRP test case set (<http://neo.lcc.uma.es/vrp/vrp-instances/capacitated-vrp-instances/>). Select 6 small, medium and large cases in Set A and Set E, and run 10 times to record the best results. Among them, BKS represents the best known solution, best is the optimal solution in the 10th time of the algorithm, and Gap represents the deviation value, that is, the deviation percentage between the optimal solution of the algorithm and the recognized optimal solution. The calculation method is  $\frac{best-BKS}{BKS} \times 100\%$ .

**Table 1.** Comparison of optimal solutions before and after the improved algorithm

case	BKS	SA		I-SA	
		best	Gap%	best	Gap%
A_36_k5	799	815	2	810	1.3
A_45_k7	1146	1170	2.1	1150	0.3
A_65_k9	1177	1255	6.7	1215	3
E_22_k4	375	375	0	375	0
E_33_k3	835	853	2.1	841	0.7
E_76_k7	682	771	13	709	3
AVG	—	—	4.3	—	1.5

It can be seen from Table 1 that the simulated annealing algorithm has a strong ability to process small data sets in a limited time, but as the number of customers in the data set increases, the optimal solution and the recognized optimal solution obtained in the limited time. The gap between them is also getting bigger and bigger. The average Gap value of the traditional SA algorithm and the known optimal solution is 4.3%, while the improved hybrid simulated annealing algorithm I-SA and the known optimal solution have an estimated Gap value of 1.5%. Under the same number of iterations, its I-SA algorithm greatly improves the quality of large data set solutions. A good balance is achieved between the solution quality of the algorithm and the calculation time. Therefore, the algorithm is more practical.

#### 3.2. Algorithm Comparison

In order to further illustrate the performance of the I-SA algorithm, the I-SA algorithm is used to calculate the test cases in the literature [2] and the literature [3] and the literature [7], and compare with the local iterative search (ILS) and the literature [2]. The experimental data in the bee colony algorithm (BCO) in [3] and the ant colony algorithm (ACO) in [7] are compared

separately, covering various experimental data sets as much as possible to prove the practicality of the algorithm in real applications The comparison results are shown in Table 2, Table 3, and Table 4.

**Table 2.** Comparison with local iterative search algorithm [2]

case	BKS	ILS		I-SA	
		best	Gap%	best	Gap%
A_36_k5	799	817	2.3	810	1.3
A_45_k7	1146	1183	3.2	1150	0.3
A_65_k9	1177	1201	2.04	1215	3
E_23_k4	375	375	0	375	0
E_33_k3	835	841	0.7	841	0.7
E_76_k7	682	721	5.72	709	3
AVG	—	—	2.3	—	1.5

**Table 3.** Comparison with bee colony algorithm [3]

case	BKS	BOC		I-SA	
		best	Gap%	best	Gap%
A_33_k5	661	678	2.57	662	0.1
A_45_k7	1146	1265	10.4	1150	0.3
A_55_k9	1073	1132	9.16	1094	1.9
A_65_k9	1177	1223	3.9	1215	3
AVG	—	—	6.5	—	1.3

**Table 4.** Comparison with ant colony algorithm [7]

case	BKS	AOC		I-SA	
		best	Gap%	best	Gap%
A_33_k5	661	780	18	662	0.1
A_45_k7	1146	1415	27	1150	0.3
A_55_k9	1073	1255	17	1094	1.9
A_65_k9	1177	1855	58	1215	3
E_101_k8	817	1413	72	876	7
AVG	—	—	38.4	—	2.5

For the 6 test cases in Table 2, the average deviation between the ILS algorithm and the Best solution is 2.3%, while the average deviation between the I-SA algorithm and the Best solution is only 1.5%; and in the 5 cases, the I-SA algorithm The best solutions are better than the ILS algorithm. For the four test cases in Table 3, the average deviation between the BOC algorithm and the Best solution is 6.5%, while the average deviation between the I-SA algorithm and the Best solution is only 1.3%. In Table 4, the average deviation between the AOC algorithm and the Best solution is 38.4%, while the average deviation between the I-SA algorithm and the Best solution is only 2.5%. By comparing the time complexity and solution quality, we can get that the stability and optimization performance of the I-SA algorithm are better than the ILS, BOC and AOC algorithms.

#### 4. Conclusion

This paper designs a hybrid simulated annealing algorithm based on sub-path local search. On the basis of the simulated annealing algorithm, the sub-path local search algorithm is introduced to enhance the local fine-tuning ability and local optimization ability of the

algorithm, improve the search efficiency of the algorithm, and achieve a good balance between the solution quality and the calculation efficiency of the algorithm. The test results on the standard case of CVRP show that the algorithm in this paper achieves a good balance between the solution result and the calculation time. It effectively avoids the disadvantage of meta-heuristic algorithm that consumes a long time, and can provide logistics enterprises with a cost-effective transportation plan that meets customer needs in a short calculation time, and has good practical application potential.

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