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Hybrid Reliability Analysis Based on Augmented Lagrangian Multiplier and Cuckoo Search Algorithm

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Abstract

A hybrid reliability analysis method is proposed for the reliability analysis of complex engineering structures. The outer layer is the minimum reliability index mathematical model, and the inner layer is the constraint condition of limit state equation. The constrained optimization problem is transformed into the unconstrained optimization problem by the augmented Lagrangian multiplier method. Then the nested cycle reliability optimization problem is solved by the proposed algorithm. Compared with the first order second moment method and Monte Carlo method, the number of iterations of the proposed method is less and its efficiency is better. Numerical simulations are provided to verify the effectiveness of the proposed algorithm in solving complex reliability problems.

Keywords

Hybrid reliability; Correlation; Augmented Lagrange multiplier method; Cuckoo search algorithm.

1. Introduction

In practical engineering, due to various complex factors such as material characteristics, boundary conditions, etc., the traditional reliability solution method has become more and more difficult to adapt to today's environment, so the solution method of mixed reliability problem is particularly important. There are many traditional reliability analysis methods, including first-order second moment method, Monte Carlo method and so on. However, the strong dependence of these methods on data limits their application in practical engineering structures. Many advanced algorithms have been developed [1-6], especially in the reliability analysis. Li et al. applied the genetic algorithm for the parameter estimation of hybrid Weibull distribution, and this method improves the performance of traditional genetic algorithm [7]. Li et al. used the kriging model to replace the real limit state equation, and established a reliability calculation method based on the improved Kriging response surface [8]. Luo et al. combined the BP neural network model with the improved AdaBoost algorithm, and proposed an improved BP AdaBoost algorithm model to calculate the reliability [9]. Xia et al. combined the response surface method with the simplex optimization to solve the convergence problem of limit state equation with high nonlinearity and certain engineering practicability [10]. Through the analysis of the references mentioned above, it is found that there are a few papers which report on the analysis of correlation.

Based on this above, a hybrid algorithm is proposed in this paper based on the augmented lagrange multiplier and the cuckoo buffeting algorithm, and the proposed method is used to solve the structural reliability analysis problem with parameter uncertainty and correlation. In

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this method, the uncertain model is constructed by introducing the uncertainty and correlation of parameters. The constrained problem is transformed into the unconstrained problem by the augmented Lagrangian multiplier method, and is optimized by the cuckoo search algorithm. Compared with the penalty function method, the selection of initial penalty factor is avoided. Under the influence of a certain degree of parameter uncertainty and correlation, the proposed method can effectively solve the limit state function problem with a certain linear or nonlinear objective function, and converge at a faster speed. Numerical simulations are given to verify the stability and effectiveness of the proposed algorithm.

2. The Establishment of Reliability Index and the Introduction of Algorithm

2.1. The Establishment of Reliability Index

If the limit state equation of the actual engineering structure is expressed as $Z = g(X_1, X_2, ..., X_n) = 0$, $X_1, X_2, ..., X_n$ in the formula are independent random variables and distributed in any form. The R-F method is used to normalize the equivalent of non-normal variables, the derive mean value μ'_{xi} , standard deviation σ'_{xi} , and its reliability index β are respectively given as:

$$\sigma'_{xi} = \phi\left\{\Phi\left[F_{xi}\left(xi^*\right)\right]\right\} / f_{xi}\left(X_i^*\right) \tag{1}$$

$$\mu'_{xi} = X_i^* - \Phi^{-1}\left[F_{xi}\left(xi^*\right)\right] / \sigma'_{xi} \tag{2}$$

$$\beta = \left(\sum\left[\left(X_i^* - \mu'_{xi}\right) / \sigma'_{xi}\right]^2\right)^{1/2} \tag{3}$$

In the formula, because the initial design point is unknown, in order to calculate the minimum value of reliability index β , β should be regarded as the distance between the point $P\left(X_1, X_2, ..., X_n\right)$ and the surface of limit state equation. The minimum value of β can be obtained through the derivation and the calculation, and then the point P obtained is the design point. The mathematical model of reliability index is established as follows:

$$\begin{aligned} Min\beta^2 &= \sum_{i=1}^{n}\left[\left(X_i^* - \mu'_{xi}\right) / \sigma'_{xi}\right]^2 \\ s.t. Z &= g\left(X_1^*, X_2^*, ..., X_n^*\right) = 0 \end{aligned} \tag{4}$$

2.2. The Augmented Multiplier Method with Equality Constraints

Using the Lagrange multiplier method, the mathematical model of the constrained optimization problem of formula (2) can be transformed as:

$$\min M'\left(X, \lambda, r\right) = M\left(x\right) + \sum_{u=1}^{L}\lambda_u g_u\left(X\right) + \frac{r}{2}\sum_{u=1}^{L}\left[g_u\left(X\right)\right]^2 \tag{5}$$

Where r is the penalty factor of the external penalty function, and λ is the Lagrangianmultiplier. In Eq. (5), the former term on the right side of the equation is the multiplier term, and the latter part is the penalty term. When the equality constraint of Lagrange multiplier method is used, the penalty factor needs to choose a larger value or increase according to a certain standard. In this proposed method, both the penalty function method and the Lagrange multiplier method are used, and this proposed algorithm can avoid the selection of the initial penalty factor in the penalty function method.

2.3. The Cuckoo Search Algorithm

Animals have their own way of reproduction in nature. Cuckoos breed by laying eggs in the nests of other birds. They search for the nests that can lay eggs by random or similar random flight. In order to simulate the breeding mode of cuckoo, the following three ideal states are assumed:

(1) Each cuckoo lays only one egg at a time, and randomly selects one from the nearby nests for hatching.

(2) In the selected random nest, the excellent nest will be saved to the next time.

(3) The number of nests that can be selected nearby is fixed. The probability that the owner of each nest finds the eggof cuckoo is $Pa \in [0,1]$.When the nest owner finds that the eggs hatched are not his own, he will discard the foreign eggs or build new nests in other places.

According to the three ideal states above which are assumed, the renewal formula for cuckoo to find the nest and the path for laying eggs is given as

$$x_i^{(t+1)} = x_i^{(t)} + \alpha \otimes L(s,\lambda) \tag{6}$$

$$L(s,\lambda) = \frac{\lambda\Gamma(\lambda)\sin(\pi\lambda/2)}{\pi}\left(\frac{1}{s^{1+\lambda}}\right),(s\ 5\ \ 0) \tag{7}$$

$x_i^{(t)}$ represents the position at the t iteration; The step factor is $\alpha > 0$; The parameter values all obey the normal distribution.

The position of each iteration is updated by Eq. (6) to generate a random number r from $[0,1]$. Then we will compare the generated result r with Pa . If $r > Pa$, the nest position is randomly generated once again; if $r < Pa$, the position of the nest remains the same. Through the above iterative process, the specific implementation of the algorithm is given as follows:

(1) The initialization operation is to set the population size, search space, iteration times, etc. In the proposed algorithm, the nest position is randomly initialized as X_i , $i \in [1,n]$. We define the objective function $f(X)$, $X = [X_1, X_2, \backslash\backslash\backslash, X_n]$.

(2) The current optimal value is obtained by calculating the objective function value of each nest location.

(3) For other nests other than the optimal nests, Levy flight is used to update, recalculate the objective function value, and compare with the current optimal value obtained in the second part. If it is better, it will replace the previous optimal value.

(4) Position update completes, and compare r with Pa , if $r > Pa$, a new nest position will be generated randomly, otherwise the nest position will not change.

(5) If the maximum number of iteration is reached or the search accuracy is up to standard, it will proceed to the next step, otherwise turn back to step 3.

(6) Output the location of the global optimal nest.

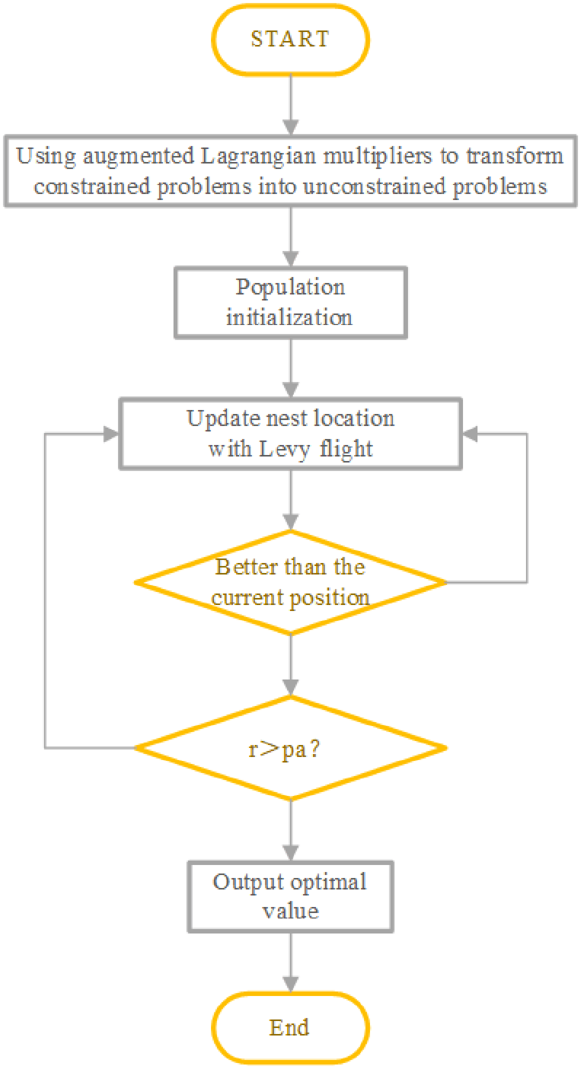

Figure 1. The flow chart of the proposed algorithm

In the present algorithm, the augmented lagrangian multiplier method and the cuckoo search algorithm are combined. Through the lagrangian augmented multiplier method, the constrained optimization mathematical model is transformed into the unconstrained optimization mathematical model, and the cuckoo search algorithm is used for the optimization, and it is applied to the calculation of reliability index, so as to analyze the reliability of the structure. The calculation flow chart 1 of the proposed algorithm is shown in Figure 1.

3. Reliability Calculation Example

3.1. Numerical Example

We consider the following limit state equation

$$g = 18.46 - 7.48 x_1 / x_2^3 \tag{8}$$

In the formula, x_1 and x_2 are independent and obey the normal distribution. Their mean value and standard deviationare respectively $\mu_1 = 10$, $\sigma_1 = 2$, $\mu_2 = 2.5$, $\sigma_2 = 0.375$. .The primary second moment algorithm and the proposed algorithm in this paper are respectively used for iterative calculation, and their comparison results are shown in Table 1.

Table 1. The computaional results of two methods

Iteration	First order second moment method β_1	The proposed method β_2
1	5.8764	2.7461
2	4.9655	0
3	4.4105	2.7461
4	3.8643	2.3276
5	3.1326	2.3276
6	2.6341	2.3276
7	2.3312	2.3276
8	2.3312	2.3276
9	2.3312	2.3276
10	2.3312	2.3276

From Table 1, we can see that the reliability index obtained by the first order second moment method is 2.3312. The reliability index obtained by the proposed algorithm in this paper is 2.3276. The results obtained by the two methods mentioned above are basically the same, which proves the effectiveness of the proposed algorithm. The comparison between the two algorithms is shown in Figure 2.

Through the comparative analysis of Table 1 and Figure 2, we can see that both algorithms mentioned above can get the stable results in a small number of iteration steps. In this example, the first and second-order moment algorithm gets the stable value in the seventh step, and the proposed algorithm in this paper gets the stable value in the forth step, which can verify the feasibility of the present algorithm. Compared with the first and second-order moment algorithm, the proposed algorithm in this paper has fewer steps and higher calculation efficiency. The direct sampling Monte Carlo method is used for the calculation and the comparison with the above two methods. We set the sampling times $M = 10^5$. The calculation results are shown in Table 2.

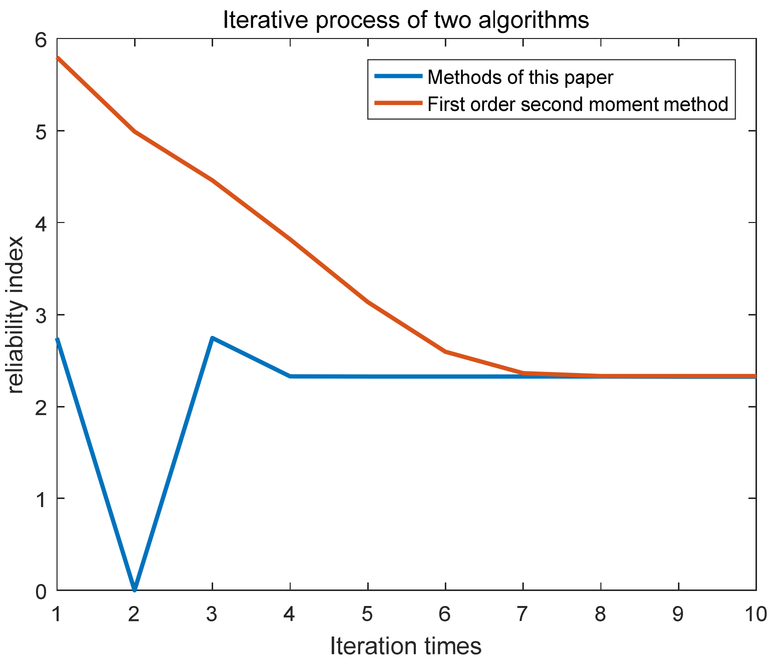

Figure 2. The iterative process of two algorithms

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Table 2. The comparison table of three methods

	The proposed algorithm	First order second moment algorithm	Monte Carlo method
Reliability index β	2.3276	2.3312	2.3300

According to Table 2, the three results are basically the same. However, the number of iterations of the proposed algorithm is less than that of Monte Carlo method and first order second moment method, which shortens the calculation time and improves the calculation efficiency. So the proposed algorithm is better. We also calculate the uncertainty of parameters μ and σ of variable x_1, x_2 with different settings. The computational results are shown in Table 3.

Table 3. The reliability index with uncertainty of relevant parameters

Uncertainyof parameters /%	β Change range	β Uncertainty /%
5	[2.2187, 2.4461]	9.1248
10	[2.1179, 2.5626]	19.0936
15	[2.0258, 2.6788]	28.4502
20	[1.9414, 2.7956]	37.9575

The relationship between the reliability index value and the uncertainty change of relevant parameters is made through the data in Table 3, as shown in Figure 3.

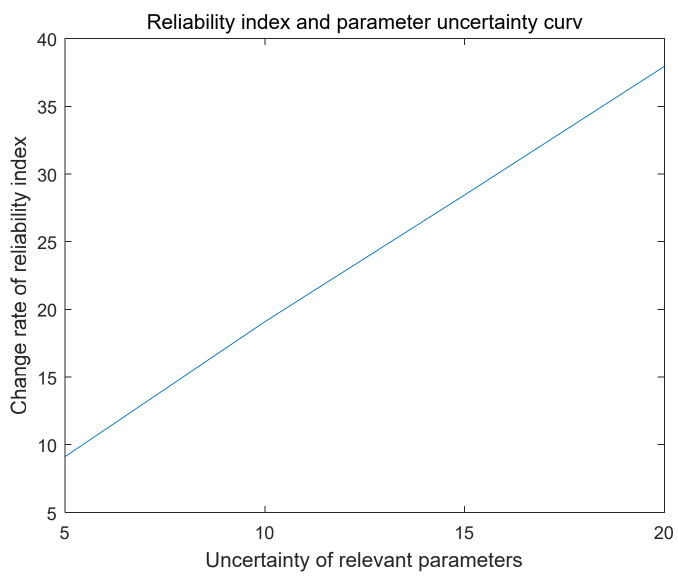

Figure 3. The curve of reliability index changing with uncertainty curve of related parameters

It can be seen in Figure 3 that the reliability index value calculated by the proposed algorithm in this paper has a positive correlation with the parameter uncertainty. When the parameter uncertainty changes, the change rate of the reliability index also increases.

3.2. Engineering Example

We consider a simple supported beam with rectangular section. Its length is l; section width is b; the height is h; the uniform load is q; the strength is R . The values of each parameter are shown in Table 4, and all the parameters follow normal distribution. The material of simply supported beam is 45 steel. The diagram of simply supported beam is shown in Figure 4, and the structure function is given as

$$Z = G(q, R, l, b, h) = R - (0.75ql^2)/(bh^2) \tag{9}$$

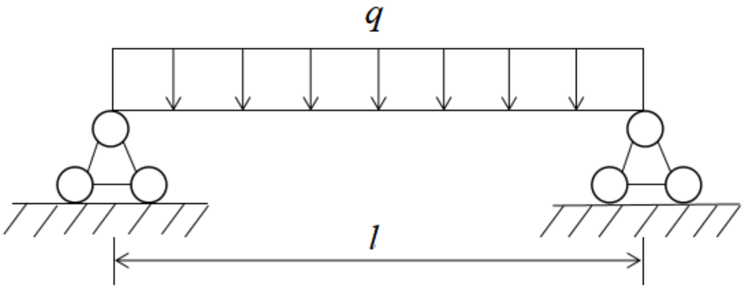

Figure 4. The simply supported beam

Table 4. The materail parameter of simply supported beam

Random variable	Mean value	Standard deviation	Distribution type
l / mm	4000	900	Normal
b / mm	120	30	Normal
h / mm	240	50	Normal
R / MPa	0.5	0.1	Normal
q / (N/mm^2)	210	50	Normal

Using the algorithm in this paper to calculate the reliability index, the results are shown in Table 5. The reliability index value is β =15.7091, and the failure probability is P_f =6.5513e-56. The iteration process of the proposed algorithm in this paper is shown in Figure 5.

Table 5. The reliability index calculation results

Reliability index	Failure probability
15.7091	6.5513e-56

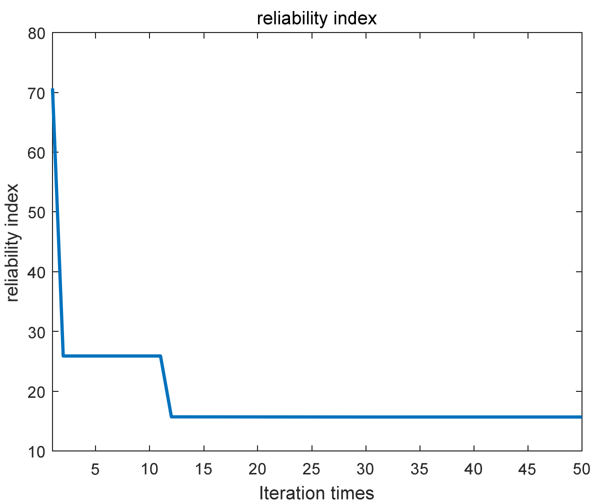

Figure 5. The iteration diagram of reliability index

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By analyzing the calculation results of Table 5 and Figure 5, the failure probability of the engineering example structure is close to zero under the given parameters in this paper, this shows that it has a high reliability. When the linear degree of the limit equation is low, the convergence speed will slow down, and the number of iteration steps by which the stable value is obtained will increase. The proposed algorithm in this paper obtains the stable reliability index value in about 12 steps.

There are many parameters involved in the calculation of actual engineering structure, and each parameter will have obvious correlation and certain interaction. In this paper, the random variable X_i and X_j are used to describe the degree of correlation coefficient is $\rho_{X_iX_j}$ [11]:

$$\rho_{X_iX_j}=\frac{E[(X_i-\mu_{X_i})(X_j-\mu_{X_j})]}{\sigma_{X_i}\sigma_{X_j}}=\frac{E(X_iX_j)-\mu_{X_i}\mu_{X_j}}{\sigma_{X_i}\sigma_{X_j}} \tag{10}$$

When we calculate the reliability index, the correlation will affect the results. Let the function be expressed as a linear function of normal random variable $x_1,\cdots,x_n$, that is:

$$Z=a_0+a_1X_1+\cdots+a_nX_n \tag{11}$$

Then the calculation formula of reliability index is:

$$\beta=\frac{a_0+a_1\mu_{X_1}+\cdots+a_n\mu_{X_n}}{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}a_i\sigma_{X_i}\rho_{X_iX_j}\sigma_{X_j}a_j}} \tag{12}$$

If the correlation between the parameters l and b is considered, then the correlation coefficient is ρ_{lb} =0.8. Other parameters are independent of each other, and the calculated structural reliability index is shown in Table 6.

Table 6. The calculation result of reliability index

Reliability index	failure probability
8.7100	1.5194e-18

In the calculation of reliability index, parameter correlation is added, and the reliability index value is calculated as β =8.7100, and this failure probability is P_f =1.5194e-16. When it is compared with no parameter correlation, the reliability index decreases, and the failure probability increases. The iteration diagram considering the variable correlation is shown in Figure 6.

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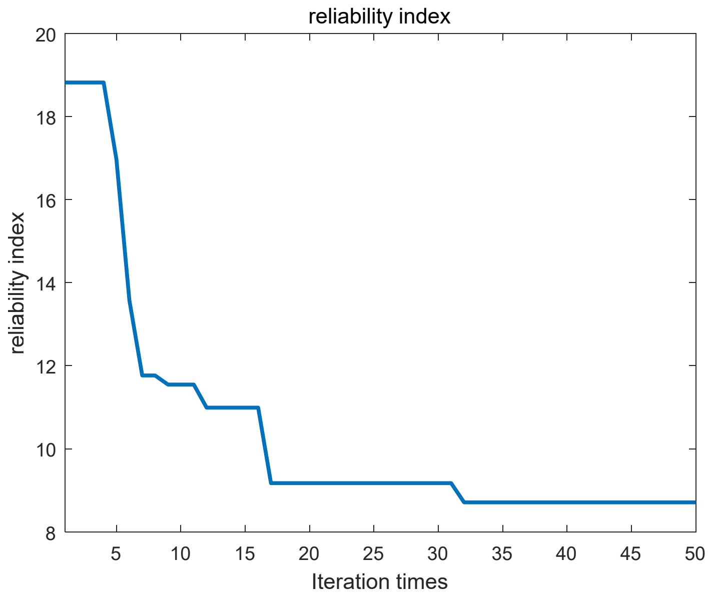

Figure 6. The iteration diagram of reliability index

By observing Figure 6, it can be seen that the relatively stable reliability index value is obtained in step 32. After the parameter correlation is added, the structural reliability will be affected. In the engineering calculation example, the addition of parameter correlation will reduce the reliability index and reduce the structural reliability.

4. Conclusion

A hybrid reliability analysis method is proposed to calculate the reliability index of engineering structure, and the results are compared with the first order second moment method and Monte Carlo method. The results are basically consistent, which verifies the feasibility of the proposed method. The proposed method can be used to solve the problem of reliability analysis of mixed reliability model with nonlinear structural function equation, parameter uncertainty and parameter variable correlation. It also has certain guiding significance for the guarantee of structural reliability.

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