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Application of Wavelet Denoising in Ultrasound Imaging

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Abstract

Ultrasound imaging has been widely used in detection, but due to the high frequency of the ultrasonic signal and the messy noise wave signal, it is difficult for ordinary filtering methods to well suppress the noise of the ultrasonic signal. This paper aims to use the wavelet denoising method to perform the Matlab simulation of wavelet decomposition and reconstruction method denoising, hard threshold denoising, and soft threshold denoising respectively. Through the simulation and comparative analysis of three wavelet noise reduction, a more suitable wavelet noise reduction method for ultrasonic signal processing is found. The main content includes: Discuss the basic theory of wavelet analysis; introduce the use principle of wavelet de-noising of decomposition and reconstruction method and threshold method, and finally compare and analyze the de-noising.

Keywords

Ultrasound imaging, hard threshold noise reduction, wavelet transform.

1. Introduction

In 1984, French geophysicist Morlet used wavelet transform for the first time when analyzing the local characteristics of seismic waves. Because of its ability to characterize the local characteristics of signals in both time and frequency domains and the characteristics of multi-resolution analysis, it is known as " Mathematical microscope". The basic idea of wavelet transform is to decompose the original signal into a series of sub-band signals with different spatial resolutions, different frequency characteristics and direction characteristics after stretching and translation. These sub-band signals have good local time domain, frequency domain, etc. Features. These features can be used to express the local features of the original signal, and then realize the localized analysis of the signal time and frequency, thereby overcoming the limitations of Fourier analysis in processing non-stationary signals and complex images. With wavelet With the maturity of the theory, people pay more and more attention to the practical application of wavelet transform. It has been widely used in signal processing, image processing, quantum field theory, seismic exploration, speech recognition and synthesis, music, radar, CT imaging, color copying , Fluid turbulence, pattern recognition, machine vision, mechanical fault diagnosis and monitoring, and digital television. In recent years, some scholars have combined wavelet transform with artificial intelligence and other theories for research, and have achieved important results. This article introduces the wavelet transform theory and applies it to the signal processing of ultrasonic testing.

2. Basic Principles of Wavelet Transform

The starting point of wavelet transform and Fourier transform is that the signal is expressed as a linear combination of basis functions. The difference is that the Fourier transform uses the harmonic function e^{inx} whose time belongs to $(-\infty,+\infty)$ as the basis function, while the basis function of wavelet transform has The generating function $\Psi(t)$ of compact support. A wavelet sequence is obtained by scaling and translating the generating function $\Psi(t)$:

$$\Psi_{a,b}(x)=\frac{1}{\sqrt{a}}\Psi(\frac{t-b}{a})$$

If a and b keep changing, we can get a family of functions $\Psi_{a,b}(t)$ from $\Psi(t)$. Given a square-integrable signal $x(t)$, namely D, the wavelet transform of $x(t)$ is

$$WT_x(a,b)=\frac{1}{\sqrt{a}}\int x(t)\Psi^*(\frac{t-b}{a})dt=\langle x(t),\Psi_{a,b}(t)\rangle$$

a is the scale factor, b is the time shift factor, and the $a^{-1/2}$ factor is the normalization constant, which is used to ensure the conservation of energy in the transformation, namely:

$$\left\|\Psi_{a,b}(t)\right\|^2=\int_{-\infty}^{\infty}\left|\Psi_{a,b}(t)\right|^2dt=\int_{-\infty}^{\infty}\left|\Psi(t)\right|^2dt$$

$\Psi(t)$ is also called mother wavelet or basic wavelet. $\Psi_{a,b}(t)$ is a family of functions produced by shifting and stretching the mother wavelet, which is called wavelet basis function, or wavelet basis for short. The function of time shift b is to determine the time position of $x(t)$ analysis, that is, the time center. The function of the scale factor a is to stretch the basic wavelet $\Psi(t)$. If $a>1$, the signal waveform shrinks; on the contrary, if $a<1$, the waveform stretches.

If t/a is replaced by t, then

$$WT_x(a,b)=\int x(t)\psi^*(\frac{t-b}{a})=\sqrt{a}\int x(at)\Psi^*(t-\frac{b}{a})dt$$

As a increases, the compressed signal waveform $x(at)$ is observed through a constant filter $\Psi(t-b/a)$. It can be seen that the scale factor a explains the changes in the scale of the signal during the transformation process. The overall signal can be observed with a large scale, and the details of the signal can be observed with a small scale.

Let the Fourier transform of $x(t)$ be $X(\Omega)$ and the Fourier transform of $\Psi(t)$ be $\Psi(\Omega)$, by the properties of Fourier transform, the Fourier transform of $\Psi_{a,b}(t)$ is

$$\Psi_{a,b}(t)=\frac{1}{\sqrt{a}}\Psi(\frac{t-b}{a})\Leftrightarrow\Psi_{a,b}(\Omega)=\sqrt{a}\Psi(a\Omega)e^{-t\Omega b}$$

The frequency domain representation of the wavelet function is

$$\Psi_{a,b}(\Omega)=\sqrt{a}\Psi(a\Omega)e^{-j\Omega b}$$

When a decreases, the time width of the wavelet function decreases, the bandwidth increases, and the center of the window of $\Psi_{a,b}(t)$ moves to the direction that $|\Omega|$ decreases. This shows that the localization of continuous wavelet transform is variable, and the time-frequency resolution is high at high frequencies, that is, it has the nature of "zooming", which is also the nature of the adaptive window we pursue.

According to Parseval's theorem, wavelet transform can be expressed as:

$$WT_{a,b}(x)=\frac{1}{2\pi}<X(\Omega),\Psi_{a,b}(\Omega)\geq\frac{\sqrt{a}}{2\pi}\int_{-\infty}^{+\infty}X(\Omega)\Psi^*(a\Omega)e^{j\Omega b}d\Omega$$

A model of a one-dimensional signal with noise can be expressed as:

$$s(i)=f(i)+\sigma\cdot e(i),(i=0,1,\cdots,n-1)$$

In the formula, $f(i)$ is the real signal, $e(i)$ is the noise, $s(i)$ Cis the noisy signal. In actual engineering, useful signals are usually low-frequency signals or relatively stable signals, and noise signals are high-frequency signals, so the denoising process can be processed as follows.

First, perform wavelet decomposition on the actual signal, select the wavelet and determine the decomposition level to be N, then the noise part is usually contained in the high frequency.

Then, threshold quantization is performed on the high frequency coefficients of wavelet decomposition.

Finally, the wavelet reconstruction is carried out according to the low-frequency coefficients of the Nth layer of wavelet decomposition and the quantized high-frequency coefficients of the 1-N layers to achieve the purpose of eliminating noise, that is, to suppress the noise of the signal and restore the real signal in the actual signal.

In general, for one-dimensional discrete signals, the high frequency part affects the first level of detail of wavelet decomposition, and the low frequency part affects the deepest and low frequency layers of wavelet decomposition.

3. Wavelet Denoising Method

3.1. De-noising by Wavelet Decomposition and Reconstruction

Suppose a signal model of noise pollution is described as:

$$S(x) = (f(x) + n_1(x)) * n_2(x)$$

In the formula, $s(x)$ is degraded signal, $f(x)$ is original signal, $n_1(x)$ is additive noise, and $n_2(x)$ is multiplicative noise. In most cases, the signal degradation process can be regarded as a linear invariant model, and the above formula can be rewritten as:

$$S(x) = f(x) + n(x)$$

$n(x)$ is Gaussian white noise. The multi-resolution analysis feature of wavelet decomposes the signal at different scales in multi-resolution, and decomposes the mixed signal composed of different frequencies into sub-signals of different frequency bands. The signal has the ability to be processed by frequency band. Because noise $n(x)$ is a real, stationary white Gaussian noise with variance σ^2, the average power of its wavelet coefficients is inversely proportional to the scale. And the amplitude of its discrete detail signal keeps decreasing with the increase of wavelet transform series. For all scales, the contrast of discrete detail signal coefficients of white noise wavelet transform will decrease regularly as the scale increases. And because the wavelet transform is a linear transform, the wavelet coefficients of the degraded signal are the sum of the wavelet coefficients of the signal and the wavelet coefficients of the noise; the discrete approximation part and the discrete detail part of the degraded signal are the discrete approximation part and discrete after the signal transformation, respectively The sum of the detail part and the discrete approximation part after noise transformation and the discrete detail part. Therefore, in the de-noising process, after the wavelet transformation of the signal and white noise, their respective wavelet coefficients have different properties, which can eliminate or reduce the noise.

Wavelet analysis is used in signal denoising processing, which is mainly manifested in the following aspects: it is for the signal to show different laws at different resolutions after wavelet transformation, set different thresholds at different resolutions, adjust wavelet coefficients, and achieve noise removal purpose.

Steps to de-noise using wavelet decomposition and reconstruction:

First, perform wavelet decomposition on the noisy signal $f(x)$ to obtain the approximation part $c_{j,k}$ and the detail part $d_{j,k}$ after wavelet transformation.

Then, take out the detail part $d_{j,k}$ of the j-th layer, and process it according to the selected threshold δ_j .Use the following formula

$$d_{j,k}=\begin{cases}d_{j,k},当 d_{j,k}>\delta_j\\ 0,当 d_{j,k}\le\delta_j\end{cases}$$

Finally, the approximation part $c_{j,k}$ and the detail part $d_{j,k}$ are used to reconstruct using the reconstruction algorithm to obtain the filtered signal.

3.2. Wavelet Transform Threshold Denoising

Since the wavelet bases of the wavelet transform are all compactly supported, the wavelet transform has a kind of "concentration" ability, which can make the energy of the signal concentrate on a few coefficients in the wavelet transform domain, so relatively speaking, the value of these coefficients It must be larger than the wavelet coefficient value of the noise whose energy is dispersed in a large number of wavelet coefficients in the wavelet coefficient domain. This means that threshold processing on the wavelet coefficients can remove noise below a fixed amplitude in the wavelet transform domain. Wavelet threshold denoising methods can be divided into hard threshold method and soft threshold method.

The processing steps of the hard threshold are as follows:

First, the signal is subjected to wavelet transform to obtain the wavelet coefficients;

Second, calculate the threshold, compare the absolute value of the wavelet coefficient with the threshold, set the points less than or equal to the threshold to zero, and keep the points greater than the threshold unchanged;

Finally, the processed wavelet coefficients are subjected to wavelet transformation to reconstruct the signal.

The processing steps of the soft threshold are as follows:

First, the noisy signal is decomposed by wavelet, the appropriate wavelet is selected, the decomposition level M of the wavelet is determined, and the signal is decomposed by dyadic discrete wavelet. The Daubichie 4 wavelet can be used, which can be realized by a correctly designed QFM (quadrature image filter).

Table 1. Daubichie 4 filter coefficients

n	1	2	3	4	5	6	7	8
h(n)	-0.0106	0.0329	0.0308	-0.187	-0.028	0.6309	0.7418	0.2304

Second, properly process the coefficients of each level after signal decomposition: select a soft threshold $\sqrt{2\log(n)}$ (n is the signal length) for the wavelet coefficients of the first to Mth levels, and perform threshold quantization on the wavelet coefficients of each level.

Finally, the signal is reconstructed: the wavelet coefficients of all levels after the quantization process are reconstructed.

The functions to get the threshold in Matlab are: ddencmp, thselect, wbmpen, wdcbm. The functions to achieve signal threshold denoising in Matlab are: wden, wdencmp, wthresh, wthcoef, wpthcoef, wpdencmp.

4. Realization and Analysis of Wavelet Denoising

The following is an example analysis of the signal generated in the ultrasonic detection. As shown in Fig. 1, the ultrasonic signal generated in the ultrasonic defect detection of the metal block, because the signal frequency is high, the signal is directly subjected to wavelet noise reduction. It is easy to analyze and compare, so a section of the signal at the bottom echo that is important in the ultrasonic signal is selected for wavelet decomposition and reconstruction

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method denoising, hard threshold denoising and soft threshold denoising. This article uses Matlab to perform wavelet denoising simulation. The following are simulations and analysis based on wavelet decomposition and reconstruction method denoising, hard threshold denoising, and soft threshold denoising.

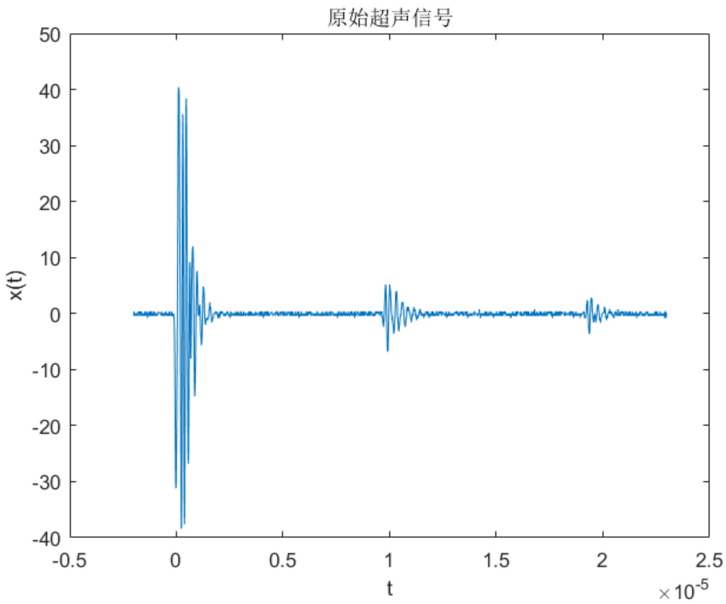

Fig 1. Ultrasonic signal generated in ultrasonic defect detection of metal block

4.1. Denoising based on Wavelet Decomposition and Reconstruction Method

Select sym8 as the wavelet base, and decompose the ultrasonic signal in 4 layers. After denoising by matlab wavelet decomposition and reconstruction method, the result is shown in Fig. 2.

As shown in Fig. 2, the original signal is a segment of the signal near the bottom echo in ultrasonic testing, and the original signal in the following figure is the same signal as this signal. It can be seen that the clutter of the ultrasonic signal is suppressed after denoising by wavelet wavelet decomposition and reconstruction method, but it is not obvious.

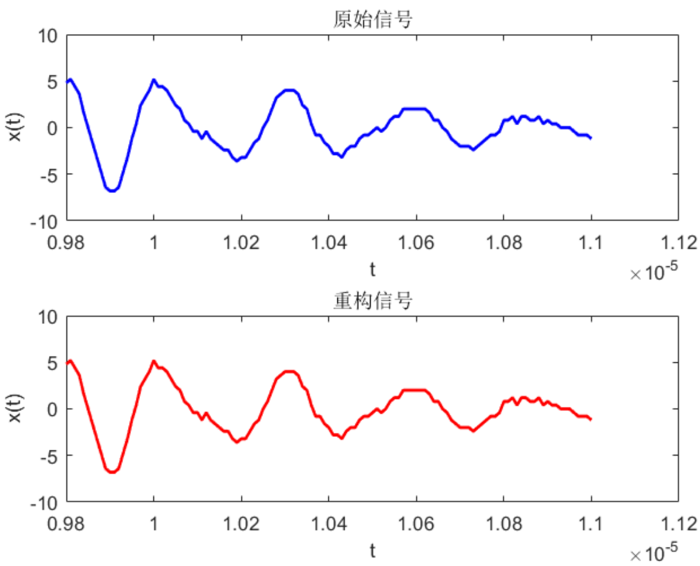

Fig 2. Denoising based on wavelet decomposition and reconstruction method

4.2. Hard Threshold Noise Reduction

Select sym6 as the wavelet base, decompose the ultrasonic signal in 4 layers, and the result after matlab wavelet hard threshold denoising is shown in Fig. 3:

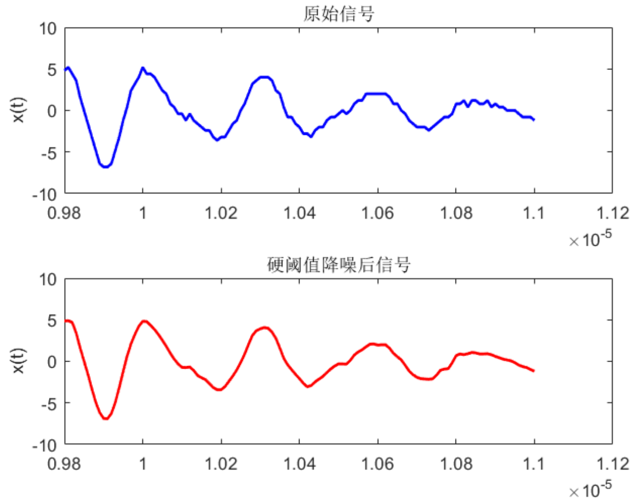

Fig 3. Wavelet Hard Threshold Denoising

As shown in Fig. 3, compared with the original signal, the signal clutter that has undergone wavelet hard threshold noise reduction is significantly suppressed, and the signal waveform becomes relatively smooth.

4.3. Soft Threshold Noise Reduction

Select sym6 as the wavelet base, and decompose the ultrasonic signal in 4 layers. The result after matlab wavelet soft threshold denoising is shown in Fig. 4:

As shown in Fig. 4, compared with the original signal, the signal clutter that has undergone wavelet soft threshold noise reduction is significantly suppressed, and the signal waveform becomes relatively smooth.

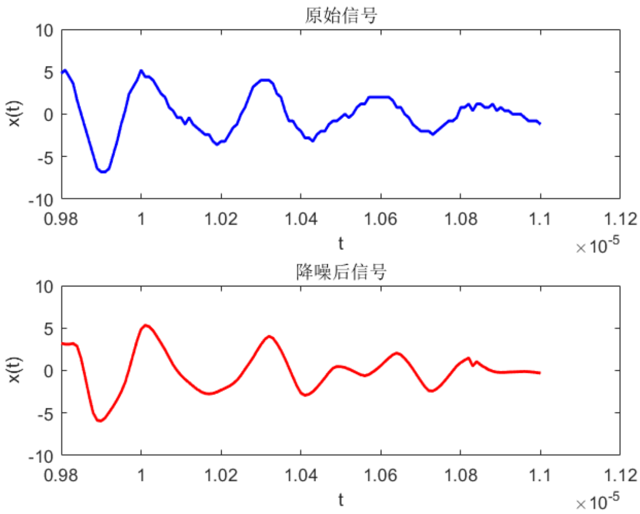

Fig 4. Soft threshold noise reduction

It can be seen from the anti-authentication results of three kinds of denoising: Denoising using wavelet decomposition and reconstruction method will force the high-frequency coefficients to 0 and then reconstruct the signal, which will lose some details of the signal. If it is image denoising It will make it blurred, and the wavelet threshold denoising method is better than the wavelet decomposition and reconstruction method. But hard threshold denoising. The absolute value of the signal is compared with the specified threshold, the point less than or equal to the threshold becomes zero.Points below the threshold remain unchanged; while soft threshold denoising, which compares the absolute value of the signal with the specified threshold, points less than or equal to the threshold becomes 0; points greater than the threshold become the difference between the point value and the threshold . Generally speaking, the hard threshold function is better than the soft threshold method in the sense of mean square error, but the signal will produce additional oscillations and jump points, which does not have the smoothness of the original signal. The overall continuity of the wavelet coefficients obtained by soft threshold estimation is good, so that the estimated signal will not produce additional oscillations, but it is better than the compressed signal, and will produce a certain deviation, which directly affects the approximation of the reconstructed signal and the real signal.

5. Summary

Wavelet transform is a signal time-frequency analysis method. It has the characteristics of multi-resolution analysis. It is very suitable for detecting the transient anomalies entrained in the normal signal and displaying its components, effectively distinguishing the sudden change in the signal and the noise. The ultrasound signal in imaging is exactly the signal with transient anomalies in this normal signal. Therefore, the use of wavelet transform can effectively denoise the ultrasound signal while extracting noise-containing signals. Using traditional Fourier transform analysis, it seems powerless, because Fourier analysis analyzes the signal completely in the frequency domain. It cannot give the change of the signal at a certain point in time, so that any sudden change of the signal on the time axis will Affect the entire spectrum of the signal. It is proved by examples that the denoising method based on wavelet transform is a superior method for extracting useful signals, displaying noise and sudden change signals, and it is very suitable for ultrasonic imaging. Besides, there are many other broad practical values.

References

[1] Wufan Chen. Wavelet analysis and its application in image processing [M]. Beijing: Science Press, 2002.

[2] Dawei Lu, Li Shu. Application of wavelet transform in real-time damage detection of laminates. Journal of Beihang University [J],2003, 29(11):1047-1050.

[3] Changhua Hu, Guohua Li, Tao Liu. System Analysis and Design Based on Matlab——Wavelet Analysis [M]. Xi'an: Xidian University Press, 2004.

[4] Zhengxing Cheng. Wavelet analysis algorithm and application [M]. Xi'an: Xidian University Press, 1998.

[5] Qiwen Ran, Liying Tan. Wavelet analysis and Fourier transform and its application [M]. Beijing: National Defense IndustryIndustry Press, 2002.

[6] Yankui Sun. Wavelet analysis and its application [M]. Beijing: Mechanical Industry Press, 2005.

[7] Zhenqing Liu, Guang Chen. Spectrum analysis technology in ultrasonic nondestructive testing. Shanghai: Non-destructive testing [J],2001,23(2):85-88.

[8] Shengyun Wan. Selection of ultrasonic testing frequency and defect waveform analysis of steel castings, casting technology [J],2003,5:441-443.

[9] Yuhua Peng. Wavelet Transform and Engineering Application [M]. Beijing: Science Press, 2002.

[10] Xiaomin Wang, Mingxuan Li. Ultrasonic testing and evaluation of composite materials. Beijing: Applied Acoustics [J],1998,17(6):107-110.