

Solving Nonlinear Parameter Estimation Problems in Measurement Data Processing Using the Fish Swarm Algorithm

Yongdi Wang^{1, a, *}

¹School of Remote Sensing and Geomatics Engineering, Nanjing University of Information Science and Technology, Nanjing 210044, China

^aydwang2003@163.com

Abstract

The artificial fish swarm algorithm is used for parameter estimation of nonlinear models to solve the traditional nonlinear group of equations with poor initial value sensitivity, poor convergence and low precision. The algorithm is very effective in solving nonlinear groups of equations, with high precision, fast convergence, strong robustness, good global convergence, and independent of the initial value selection, which not only overcomes the initial value sensitivity and poor convergence problems of traditional methods, but also does not affect the convergence and precision of arbitrary multivariate nonlinear groups of equations. The analysis of the results shows that the artificial fish swarm algorithm is feasible and effective for dam deformation prediction.

Keywords

Artificial fish swarm algorithm; Group of equations solution; Parameter estimation.

1. Introduction

With the development of statistics and science and technology, more and more nonlinear models appear in various fields, which requires active research and deep practice in parameter estimation of nonlinear models; the solution of nonlinear equations [1] has also received great attention in the surveying and mapping industry. The study of numerical methods for solving nonlinear sets of equations is of great significance, and the methods have important applications in real life.

In the research of narrowing the gap between the linear approximation and its true value and improving the accuracy of parameter estimation, many experts and scholars have researched various nonlinear parameter estimation methods, such as the trust domain method, Newton method, Gaussian-Newton method, damping least squares method, fastest descent method, direct solution method without iteration, etc. The more common method is the Gaussian-Newton method, the principle of which is to approximate the nonlinear model by linear least squares estimation, and then iteratively calculate the parameter estimation. The more common method is the Gaussian-Newton method, which uses linear least squares estimation to approximate the least squares estimation of a nonlinear model, and then performs iterative calculations to estimate the parameters, i.e., it expands to the number of stages, and takes the first term, omitting the second term and so on. This linear approximation not only causes model substitution errors, but also leads to other problems [2], such as: nonlinear parameter estimation methods may be difficult to derive due to complex functions; nonlinear iterative solutions are sometimes dependent on the choice of initial values; the normal coefficients are difficult to solve when the matrix is rank deficient or has a path matrix; inappropriate setting of initial values may cause the iterations to fail to converge, and so on. With the development of measurement technology, more complex nonlinear models are indispensable, so we have to use some new methods to estimate the parameters of nonlinear models.

In recent years, some new methods have emerged to solve the problem of parameter estimation for nonlinear models, such as the simplex method [3], simulated annealing algorithm [9], genetic algorithm [4], etc. These new intelligent algorithms have the advantage of no derivation compared to the traditional numerical methods, but there are also disadvantages. Compared with the traditional numerical methods, these new intelligent algorithms have the advantage of no derivation, but the disadvantages are also present: the simplex method does not require derivation, but the result is only a local optimum [2], so the computational effect is not very good; the simulated annealing algorithm avoids the local minimum problem, but its convergence rate will be very slow, although it can obtain a local optimum solution, resulting in the search time is too long and slow [4]. Genetic algorithms are able to fully search many points of the solution space at the same time, so they are very fast in terms of global convergence [4], avoiding the problem of falling into a local optimum, with the disadvantage that they are difficult to encode and adapt to the design of the degree function, and require high performance of the processor hardware and software.

The Artificial Fish Swarm Algorithm (AFSA), proposed by Xiaolei Li of Zhejiang University in 2002[5], is an intelligent optimization algorithm based on animal behavior, inspired by the survival behavior of fish, and is a typical application of artificial intelligence. The basic principle is that in the waters where fish live, in order to find a place with more nutrients, the fish will follow other companions by themselves or on their own. Therefore, where the number of fish is the highest, the waters have the most nutrients accordingly. The artificial fish swarm algorithm is based on this feature to construct artificial fish to mimic the clustering, foraging, random, and tail-chasing behaviors of the fish, and to find the optimal solution for a particular problem. The four behaviors of fish, clustering, foraging, tail-chasing, and random, can be applied to the problem of finding the best fish.

After nearly a decade of development, the artificial fish swarm algorithm is currently in the basic stage of theoretical research, but it shows good integration with intelligent algorithms such as simulated annealing algorithm, genetic algorithm, ant colony algorithm, particle swarm algorithm and other intelligent algorithms, and it shows good applicability in the field of communication, signal and image processing, neural networks, data mining, control field, numerical computing, power system field, agricultural water conservancy, combinatorial optimization problem and parameter optimization detection.

In this paper, we propose to apply a new algorithm, Artificial Fish Swarm Algorithm (AFSA) [6], to the solution of arbitrary multivariate nonlinear groups of equations [7-8]. The new algorithm is designed to solve the problem that linear approximation is not possible due to the complexity of the model, and in some cases the iterations cannot converge due to inappropriate selection of initial values. It is hoped that the new algorithm can compute the approximate solutions of nonlinear systems of equations in a short period of time and provide an evolutionary approach to solving nonlinear systems of equations without the need for derivatives.

2. Theory of Parameter Estimation for Nonlinear Models

There are a large number of nonlinear parameter estimation problems in various fields of natural sciences, economics, and engineering computing. Theoretically, the theory and methods of linear model estimation can also be applied to the parameter estimation of nonlinear models, but the complex relationship of nonlinear models makes them different from linear models in terms of estimation criteria and quality, as well as evaluation of parameter solving. The theory of parameter estimation for nonlinear models.

Parameter estimation is a basic form of statistical inference and is an important branch of mathematical statistics. Due to the nature of the statistical distribution of some subsamples of observations, a function estimation model can be constructed for their parameters, and a

certain numerical computation method can be used to calculate the estimates of the parent parameters under the constraints of a certain estimation criterion function. Therefore, there are three key steps in the parameter estimation process:

- a) to model the function between the estimated parameters and the subsample of observations, which should be in accordance with the relevant statistical rules;
- b) to select the appropriate estimation criterion function according to the evaluation objectives;
- c) to adopt an appropriate numerical solution method to ensure the stability and accuracy of the estimated parameter solutions.

The estimation criteria for nonlinear model parameters are similar to those for linear models, including least squares estimation, maximum likelihood estimation, absolute and minimum estimation, minimum mean square estimation, and Bayesian risk minimization. The most widely used is the nonlinear least squares (LS) estimation criterion, and the following is the existence theorem and definition of nonlinear least squares.

Set the nonlinear model as.

$$L = f(X) + \Delta \quad (1)$$

Where L is the random subsample of observations, $f(X)$ is the nonlinear model, X is the parameter to be evaluated, and Δ is the unobservable random error. The corresponding stochastic model is:

$$D_{LL} = \sigma^2 P^{-1} = \sigma^2 Q_{LL} \quad (2)$$

Where σ^2 is the unit-weighted variance factor, and P, Q are the weighted and covariant arrays of the observation vector, respectively. Since Δ is not estimable, the error equation is often used instead of the (1) equation in parameter estimation, with

$$V = f(\hat{X}) - L \quad (3)$$

V is the vector of residuals of the observed subsamples. The squared sum of the residuals is then:

$$V^T V = \|V\|^2 = \left\| f(\hat{X}) - L \right\|^2 = (f(\hat{X}) - L)^T (f(\hat{X}) - L) \quad (4)$$

Then by the least squares definition, if (4) satisfies the relation

$$V^T V = \|V\|^2 = (f(\hat{X}) - L)^T (f(\hat{X}) - L) = \min \quad (5)$$

Then say that \hat{X} is a nonlinear least-squares estimate of X .

3. Introduction to Artificial Fish Population Algorithms

3.1. Fundamentals of Artificial Fish Swarming Algorithms

The basic principle of the Artificial Fish Population Algorithm is that in the waters where the fish live, in order to find the place with the most nutrients, the fish will follow other companions, either by themselves or on their own. Therefore, where there is the largest number of fish, there is the most nutrients in the water accordingly. The artificial fish swarm algorithm is based on this feature to construct artificial fish to mimic the clustering, foraging, random, and tail-chasing behaviors of the fish, and to find the optimal solution for a particular problem. Four types of fish behaviors, clustering, foraging, tail-chasing, and random, can be applied to solve the problem. The task of the researcher is to understand the basic principles of the algorithm and use them to construct a simulation of these four basic behaviors. The foraging behavior, in which the fish swim toward the water with more nutrients, corresponds to an iterative way of moving in a better direction in an optimization algorithm, similar to the visual concept of fish swarming patterns. In herding behavior, fish spontaneously gather in groups to find food and to avoid predators. Tail-following is the act of following the most active person in the vicinity, corresponding to an optimization algorithm that moves toward the best nearby partner.

3.2. Optimization Principle of Artificial Fish Swarm Algorithm

In the artificial fish swarm algorithm, each of the four basic behaviors of the fish plays an important role in the implementation of the algorithm. During the search for optimization, the artificial fish tend to cluster around the local extreme domain. This allows the fish to escape from the local extreme domain. The crowding factor in the algorithm limits the size of the cluster, and the fish will only tend to cluster around the global extremes, which makes it easier for the fish to escape from the local extremes.

As the objective function and partners change, each fish will choose the appropriate behavior among the four basic behaviors and perform it according to its current environmental situation, and eventually the fish will congregate around local extremes.

Depending on the nature of the problem to be solved, the artificial fish will evaluate the current environment and choose a behavior, i.e., the artificial fish will simulate performing foraging, clustering, tail chasing, etc., and then evaluate the value of the specific behavior, and the optimal behavior will be performed. The termination of the algorithm is determined by the actual situation, usually by determining whether the mean squared error of the values obtained in multiple iterations is less than the allowed error, or by determining whether the number of artificial fish clustering in a particular area has reached a certain ratio, or by limiting the number of iterations of the algorithm. We can introduce a bulletin board in the algorithm to record the optimal state of the artificial fish. After each iteration, the bulletin board compares its own state with the bulletin board state, and if its own state is better than the bulletin board state, it updates the bulletin board and writes its own state to the bulletin board, so the bulletin board always records the constantly updated optimal state. After the iteration of the algorithm is completed, the record on the bulletin board is the optimal search value.

3.3. Analysis of the Effect of Each Parameter on the Convergence of Artificial Fish Population Algorithms

The artificial fish swarm algorithm has five basic parameters that play an important role in the convergence of the algorithm. Visual is the field of view of the artificial fish, Step is the maximum moving step of the artificial fish, N is the total number of artificial fish in the fish population, Try_number is the number of attempts in the execution of the algorithm, and is the crowding factor.

(a) Visual field of view

Since the choice of the field of view has a strong influence on the convergence of the algorithm, the artificial fish performs its actions within the field of view. When the field of view is small, the search ability of the artificial fish is strong; when the field of view is large, the search ability is weak. In general, the larger the field of view, the easier it is for the fish to find global extremes and converge.

(b) Step size

The choice of step size has a great impact on the convergence and speed of the algorithm, and by choosing a larger step size, the artificial fish can quickly converge to the extreme point and the speed of convergence is improved to a certain extent. However, there is a limit to the increase in step size, and after exceeding the limit, the convergence speed will slow down, and the oscillations that sometimes occur will have a great impact on the convergence speed. In the later stages of convergence, the artificial fish tends to oscillate back and forth near the global extremes, greatly affecting the speed of convergence. When small steps are chosen, the speed of convergence is slow, although the accuracy of the search can be improved.

(c) Total number of artifacts

We can increase the total number of artificial fish to make the population intelligence of the fish more salient, and at the same time, the convergence rate will be faster and the search for the best value will be more accurate. The disadvantage is that the algorithm will be too computationally intensive in each iteration. So for a particular optimization problem, we can reduce the number of artifacts appropriately, while the algorithm should be sufficiently stable.

(d) Number of attempts

The more the number of attempts, the more the fish tend to perform foraging behavior and the more efficient convergence will be, but when the local extremes are more prominent, the fish tend to cluster around the local extremes and miss the global extremes. Therefore, when optimizing the general problem, the number of attempts can be increased to improve the convergence rate; in addition, when the local extremes are more prominent, the number of attempts can be decreased accordingly to increase the chance for the fish to swim randomly, so as to reduce the influence of local extremes on the search for excellence. The more attempts, the more difficult it is for the fish to get rid of the local extremes, but when the local extremes are not very prominent, the number of attempts can be increased to reduce the probability of random fish swimming and the convergence efficiency can be improved.

(e) Crowding factor

The introduction of the crowding factor avoids overcrowding of artificial fish into local extremes. In addition, this parameter can cause the artificial fish near the extreme point to repel each other and affect the precision of the fish approaching the extreme point.

Therefore, the rational configuration of each parameter of the artificial fish is not invariable, but needs to be chosen according to different search functions and search accuracy.

4. Artificial Fish Population Algorithm for Nonlinear Parameter Estimation

4.1. Modeling

We assume that $f_i(X) = A_i (i = 1, 2, \dots, n)$ is a multivariate equation, $X = [x_1, x_2, \dots, x_n]$ is the unknown vector of the group of equations, and $A_i (i = 1, 2, \dots, n)$ is the constant term of the group of equations, creating a group of equations with n unknowns, as shown in Eq. (6).

$$\begin{cases} f_1(x_1, x_2, \dots, x_n) = A_1 \\ f_2(x_1, x_2, \dots, x_n) = A_2 \\ \vdots \\ \vdots \\ f_n(x_1, x_2, \dots, x_n) = A_n \end{cases} \quad (6)$$

Changing the form of the group of equations such that $F(X) = \sum_{i=1}^n |f_i(X) - A_i|$, the problem of finding the roots of the aforementioned multivariate nonlinear group of equations is transformed into the optimization problem of finding X to minimize the value of $F(X)$.

4.2. Relevant Symbol Definitions

Set the vector X is the state of a single artificial fish, X_i is the position of artificial fish i ;

$F(X) = \sum_{i=1}^n |f_i(X) - A_i|$ is the adaptive or rational function value at position X , which can

represent food concentration; $d_{ij} = \|X_i - X_j\|$ represents the distance between artificial fish i

and j ; $Visual$ and δ represent the visual distance and crowding factor of the artificial fish, respectively; $step$ is the maximum step of the artificial fish's movement;

$S = \{X_j \mid \|X_i - X_j\| < Visual\}$ is the search condition at the current position of the artificial fish.

4.3. Description of Behavior

a) Random behavior

Assume that the current state of the artificial fish i is X_i , and that X_i is the optimal state in the current population, then its next state is $X_{i/next}$; otherwise, $X_{i/next} = X_i + Random(-step, step)$, where $Random = (-step, step)$ is an n -dimensional random vector consisting of random numbers on the real interval $(-step, step)$, and $Step$ is the maximum moving step of the artificial fish, as follows.

b) Foraging behavior

Assume that the current state of artificial fish i is X_i , and randomly select a state $X_j = X_i + Visual \cdot Rand$ within its field of view, where $Rand$ is a random number between 0 and 1. Substitute X_i and X_j into the food concentration function. If $F(X_j) < F(X_i)$, then $X_{i/next} = X_j$ indicates that the randomly selected state X_j is better than state X_i , so the randomly selected state X_j is substituted for the current state X_i . If $F(X_j) \geq F(X_i)$, it selects a new state within its field of view and repeats the above.

c) Clustering behavior

Suppose the current state of the artificial fish i is X_i , the central position of all partners in the group of artificial fish in the field of view is X_c , and nf is the number of partners in the field of view, if $F(X_c) < F(X_i)$, then the central position X_c is better than the current state X_i then if $nf / N < \delta$ or $d_{i,c} < Visual / 2$ ($d_{i,c}$ is the distance between X_i and X_c), as long as one of these

two conditions is met, then $X_{i/next} = X_i + Random(0, step) \times \frac{X_c - X_i}{d_{i,c}}$; otherwise, the foraging behavior is performed.

d) Tailgating behavior

Fish i is X_i , the optimal state of the fish X_{min} within the field of view, and nf is the number of partners around the optimal artificial fish. Substituting X_i and X_{min} into the food concentration function, if $F(X_{min}) < F(X_i)$, then X_{min} is better than the current state X_i , then for $nf / N < \delta$ or $d_{i,min} < Visual / 2$ ($d_{i,c}$ is the distance between X_i and X_{min}), if one of the two conditions is met, then $X_{i/next} + Random(0, step) \times \frac{X_{min} - X_i}{d_{i,min}}$ is executed; otherwise, foraging is executed.

4.4. Algorithm Implementation Steps

- a) Step 1: Initialize the artificial fish population: enter the field of view $Visual$, step size $Step$, tolerance η , crowding factor δ , number of attempts Try_number , and number of artificial fish N .
- b) Step 2: Identify the artificial fish that are in the optimal state among the former group of N artificial fish and enter their information on the bulletin board.
- c) Step 3: Substitute the current optimal artificial fish state X_0 recorded on the bulletin board into the food concentration function, and see if it satisfies $F(X_0) < \eta$. If it satisfies, then go to the later stage; otherwise, go back to the earlier stage and re-optimize.
- d) Step 4: When the error of the calculation result is less than the error set in the initialization stage of the artificial fish, the bulletin board record X_0 can be output, and the algorithm ends here; if it is greater than the set error, then proceed to the next step.
- e) Step 5: Define the foraging and random behaviors of the artificial fish in the algorithm subroutine and have each artificial fish perform both behaviors, where the optimal behavior is selected and executed. where neither the clustering behavior nor the tail-chasing behavior is executed.
- f) Step 6: Compare the optimal state of the artificial fish in the current population with the record on the bulletin board, and if it is better than the record on the bulletin board, go back to step 3 and repeat the cycle.

4.5. Segment Optimization

The moving step and field of view in the parameters of the artificial fish swarm algorithm can affect the convergence speed and the accuracy of the solution value. If the field of view and step length are set too large, the search range of the artificial fish is enlarged and its swimming speed is increased, and the optimal solution can be found quickly, but the stability of the algorithm is affected, and the convergence speed is slow, so the final solution accuracy is not high; if the field of view and step length are set small, the search ability of the artificial fish is greatly reduced, and although the solution accuracy can be improved, the convergence speed is extremely slow. In order to solve this problem without affecting the convergence of the algorithm and to ensure the accuracy of the results, we can subdivide the algorithm's execution process, and set the moving step and visual distance at the early stage of the algorithm to be larger in order to get a relatively better result quickly. Accuracy is further improved. By using the segmented optimization method, we can perform a fast search at the early stage and also at the later stage of the algorithm, which not only ensures the stability of the algorithm, but also improves the accuracy of the solution.

By applying the segmented optimization method to the solution of multivariate nonlinear group of equations, we can divide the basic artificial fish swarm algorithm into two stages, which overcomes the problem of reduced accuracy in the solution of multivariate nonlinear group of equations due to the large number of variables.

5. Case Analysis

To demonstrate the usability of the artificial fish swarm algorithm in solving multivariate nonlinear groups of equations. Example 2-1-2 in "Parameter Estimation Theory and Applications of Nonlinear Models" by Xinzhou Wang [2], page 32, is chosen as the example. In this example, the nonlinear model is $L_i = x_1^2 / i + x_2$, the true values of parameters x_1 and x_2 are $X=(5.420136187, -0.25436189)'$, and the five true values of L_i and the corresponding five independent observations with the same precision are shown in Table 1.

Table 1. True and observed values of L_i

i	1	2	3	4	5
True	29.123514	14.434576	9.538264	7.090107	5.621213
Observed	29.12	14.43	9.53	7.09	5.62

Table 2. Artificial Fish Swarm Algorithm Solving Results

Number	NUM	X1	X2	TIME	Error
1	28	5.4196	-0.2537	2.58	0.0124
2	26	5.4197	-0.2531	2.32	0.0129
3	14	5.4196	-0.2530	1.15	0.0132
4	22	5.4206	-0.2614	1.88	0.0146
5	15	5.4200	-0.2563	1.28	0.0110
6	19	5.4191	-0.2526	1.58	0.0146
7	24	5.4202	-0.2593	2.18	0.0133
8	20	5.4202	-0.2585	1.69	0.0118
9	17	5.4196	-0.2552	1.41	0.0124
10	16	5.4196	-0.2557	1.31	0.0127
11	6	5.4193	-0.2535	0.43	0.0133
12	15	5.4195	-0.2543	1.23	0.0119
13	25	5.4199	-0.2562	2.32	0.0112
14	13	5.4198	-0.2559	1.05	0.0120
15	17	5.4201	-0.2592	1.44	0.0149
16	14	5.4200	-0.2569	1.22	0.0114
17	18	5.4200	-0.2569	1.48	0.0113
18	21	5.4202	-0.2558	1.74	0.0144
19	18	5.4194	-0.2531	1.55	0.0133
20	10	5.4193	-0.2550	0.81	0.0150
Avg.	18	5.4197	-0.2558	1.53	0.0128

The above procedure was performed 20 times, the results were recorded and the average value was calculated, and Table 2 shows that the maximum number of iterations was 28, the

minimum value was 6, and the average number of iterations was 18. The longest calculation took 2.58 seconds, the shortest was 0.43 seconds, and the average time was 1.53 seconds. x_1 had a solution value of 5.4197 and an absolute value of 0.0004, x_2 had a solution value of -0.2558 and an absolute value of 0.0015, both of which were small errors. The maximum value of the error of the final food concentration function is 0.0150, the minimum value is 0.0110, and the average error is 0.0128, which is an acceptable result based on the allowable error of 0.015. Since the allowable error is set to 0.015, the accuracy of the solution will be higher when the allowable error is reduced. However, the disadvantage is that the rate of convergence of the algorithm decreases, which increases the computation time. In addition, changing the field of view, the step size, and the number of artificial fish, N , can make the algorithm more reasonable. Referring to the above procedure, we set the field of view to 2 and the step size to 0.3 at the early stage of the algorithm to get a relatively better solution quickly with a larger step size and field of view, and reduce the field of view to 0.1 and the step size to 0.01 at the later stage to make the calculation result more accurate. This not only saves computation time, but also guarantees the accuracy requirement.

The above algorithm for solving multivariate nonlinear group of equations for artificial fish is valid for any multivariate nonlinear group of equations, and it is only necessary to input the unknown m of the group of equations at the initialization stage of the algorithm, then change the group of equations into a food concentration function, input it into the required subroutines of the algorithm, and change the parameters of the algorithm to obtain a satisfactory solution. Finally, we compare the artificial fish population algorithm with the annealing algorithm. The absolute values of the solution and truth errors of the annealing algorithm are 0.0025 and 0.0131, compared to 0.0004 and 0.0015 for the artificial fish swarm algorithm.

6. Conclusion

A nonlinear model parameter estimation model based on the artificial fish swarm algorithm was developed, tested using real examples, and the computational results of the artificial fish swarm algorithm were compared with the simulated annealing algorithm, and the results show that the artificial fish swarm algorithm has a higher solution accuracy. For equations of different forms, differentiable, continuous or complex, the accuracy and rate of their solution are not affected. This shows that the application of the artificial fish swarm algorithm to the parameter estimation of nonlinear models is successful and effective.

As a new type of evolutionary algorithm, artificial fish swarm algorithm is still in its infancy and has not received as much attention from scholars as ant colony and genetic algorithm, but its applications in function optimization, combinatorial problem solving, and communication fields have been deepening. In addition, the processing of measurement data in surveying and mapping disciplines requires the use of nonlinear parameter estimation, which can be greatly improved by using the artificial fish swarm algorithm to improve its solution accuracy. Through more experts and scholars to improve it and integrate it with other algorithms, its development space will be larger and play a greater role in the development of science and technology.

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